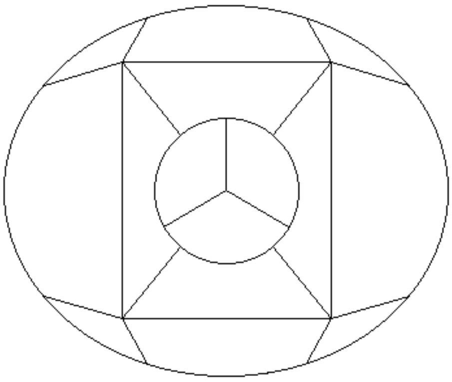
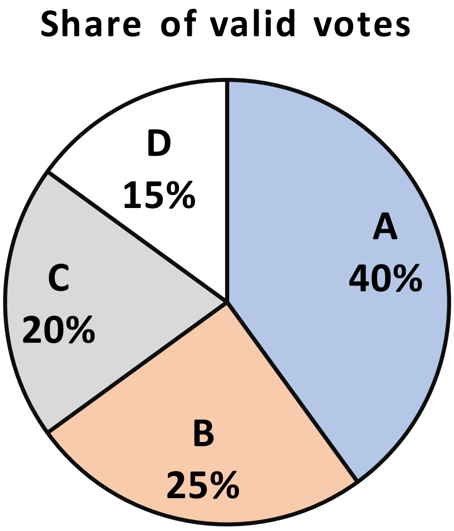
General Aptitude (GA)

Q.1 – Q.5 Carry ONE mark Each

Q.1	If ‘→’ denotes increasing order of intensity, then the meaning of the words [sick → infirm → moribund] is analogous to [silly → _______ → daft]. Which one of the given options is appropriate to fill the blank?
(A)	frown
(B)	fawn
(C)	vein
(D)	vain

Q.2	The 15 parts of the given figure are to be painted such that no two adjacent parts with shared boundaries (excluding corners) have the same color. The minimum number of colors required is 
(A)	4
(B)	3
(C)	5
(D)	6

Q.3	How many 4-digit positive integers divisible by 3 can be formed using only the digits $\{1,3,4,6,7\}$, such that no digit appears more than once in a number?
(A)	24
(B)	48
(C)	72
(D)	12
Q.4	The sum of the following infinite series is $$2+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{8}+\frac{1}{9}+\frac{1}{16}+\frac{1}{27}+\cdots$$
(A)	11/3
(B)	7/2
(C)	13/4
(D)	9/2

Q.5	In an election, the share of valid votes received by the four candidates A, B, C, and D is represented by the pie chart shown. The total number of votes cast in the election were 1,15,000, out of which 5,000 were invalid.  Based on the data provided, the total number of valid votes received by the candidates B and C is
(A)	45,000
(B)	49,500
(C)	51,750
(D)	54,000

Q.6 – Q.10 Carry TWO marks Each

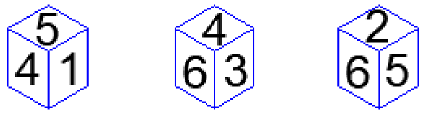
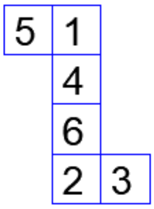
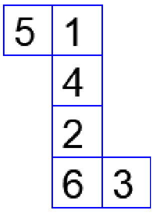
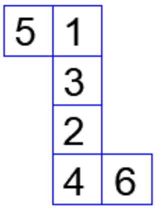
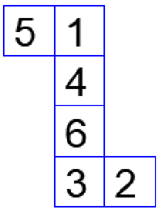
Q.6	Thousands of years ago, some people began dairy farming. This coincided with a number of mutations in a particular gene that resulted in these people developing the ability to digest dairy milk. Based on the given passage, which of the following can be inferred?
(A)	All human beings can digest dairy milk.
(B)	No human being can digest dairy milk.
(C)	Digestion of dairy milk is essential for human beings.
(D)	In human beings, digestion of dairy milk resulted from a mutated gene.
Q.7	The probability of a boy or a girl being born is 1/2. For a family having only three children, what is the probability of having two girls and one boy?
(A)	3/8
(B)	1/8
(C)	1/4
(D)	1/2

Q.8	Person 1 and Person 2 invest in three mutual funds A, B, and C. The amounts they invest in each of these mutual funds are given in the table.

	Mutual fund A	Mutual fund B	Mutual fund C
Person 1	₹10,000	₹20,000	₹20,000
Person 2	₹20,000	₹15,000	₹15,000

At the end of one year, the total amount that Person 1 gets is ₹500 more than Person 2. The annual rate of return for the mutual funds B and C is 15% each. What is the annual rate of return for the mutual fund A?

(A)	7.5%
(B)	10%
(C)	15%
(D)	20%

Q.9	Three different views of a dice are shown in the figure below.  The piece of paper that can be folded to make this dice is
(A)	
(B)	
(C)	
(D)	

Q.10	Visualize two identical right circular cones such that one is inverted over the other and they share a common circular base. If a cutting plane passes through the vertices of the assembled cones, what shape does the outer boundary of the resulting cross-section make?
(A)	A rhombus
(B)	A triangle
(C)	An ellipse
(D)	A hexagon

Q.11 – Q.35 Carry ONE mark Each

Q.11	Consider the following statements: (i) The mean and variance of a Poisson random variable are equal. (ii) For a standard normal random variable, the mean is zero and the variance is one. Which **ONE** of the following options is **correct**?
(A)	Both (i) and (ii) are true
(B)	(i) is true and (ii) is false
(C)	(ii) is true and (i) is false
(D)	Both (i) and (ii) are false

Q.12	Three fair coins are tossed independently. T is the event that two or more tosses result in heads. S is the event that two or more tosses result in tails. What is the probability of the event $T \cap S$?
(A)	0
(B)	0.5
(C)	0.25
(D)	1

Q.13	Consider the matrix $\boldsymbol{M} = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$. Which **ONE** of the following statements is **TRUE**?
(A)	The eigenvalues of $\boldsymbol{M}$ are non-negative and real.
(B)	The eigenvalues of $\boldsymbol{M}$ are complex conjugate pairs.
(C)	One eigenvalue of $\boldsymbol{M}$ is positive and real, and another eigenvalue of $\boldsymbol{M}$ is zero.
(D)	One eigenvalue of $\boldsymbol{M}$ is non-negative and real, and another eigenvalue of $\boldsymbol{M}$ is negative and real.

Q.14	Consider performing depth-first search (DFS) on an undirected and unweighted graph G starting at vertex s. For any vertex u in G, $d[u]$ is the length of the shortest path from s to u. Let (u, v) be an edge in G such that $d[u] < d[v]$. If the edge (u, v) is explored first in the direction from u to v during the above DFS, then (u, v) becomes a ______ edge.
(A)	tree
(B)	cross
(C)	back
(D)	gray

Q.15	For any twice differentiable function $f: \mathbb{R} \rightarrow \mathbb{R}$, if at some $x^* \in \mathbb{R}$, $f'(x^*) = 0$ and $f''(x^*) > 0$, then the function f necessarily has a ______ at $x = x^*$. **Note**: $\mathbb{R}$ denotes the set of real numbers.
(A)	local minimum
(B)	global minimum
(C)	local maximum
(D)	global maximum

Q.16	Match the items in **Column 1** with the items in **Column 2** in the following table:

Column 1	**Column 2**
(p) First In First Out	(i) Stacks
(q) Lookup Operation	(ii) Queues
(r) Last In First Out	(iii) Hash Tables

(A)	(p) – (ii), (q) – (iii), (r) – (i)
(B)	(p) – (ii), (q) – (i), (r) – (iii)
(C)	(p) – (i), (q) – (ii), (r) – (iii)
(D)	(p) – (i), (q) – (iii), (r) – (ii)

Q.17	Consider the dataset with six datapoints: $\{(\boldsymbol{x_1}, \boldsymbol{y_1}), (\boldsymbol{x_2}, \boldsymbol{y_2}), \dots, (\boldsymbol{x_6}, \boldsymbol{y_6})\}$, where $\boldsymbol{x_1} = \begin{bmatrix}1\\0\end{bmatrix}$, $\boldsymbol{x_2} = \begin{bmatrix}0\\1\end{bmatrix}$, $\boldsymbol{x_3} = \begin{bmatrix}0\\-1\end{bmatrix}$, $\boldsymbol{x_4} = \begin{bmatrix}-1\\0\end{bmatrix}$, $\boldsymbol{x_5} = \begin{bmatrix}2\\2\end{bmatrix}$, $\boldsymbol{x_6} = \begin{bmatrix}-2\\-2\end{bmatrix}$ and the labels are given by $\boldsymbol{y_1} = \boldsymbol{y_2} = \boldsymbol{y_5} = 1$, and $\boldsymbol{y_3} = \boldsymbol{y_4} = \boldsymbol{y_6} = -1$. A hard margin linear support vector machine is trained on the above dataset. Which **ONE** of the following sets is a possible set of support vectors?
(A)	$\{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_5}\}$
(B)	$\{\boldsymbol{x_3}, \boldsymbol{x_4}, \boldsymbol{x_5}\}$
(C)	$\{\boldsymbol{x_4}, \boldsymbol{x_5}\}$
(D)	$\{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{x_4}\}$

Q.18	Match the items in **Column 1** with the items in **Column 2** in the following table:
(A)	(p) − (iii), (q) − (i), (r) − (ii)
(B)	(p) − (ii), (q) − (i), (r) − (iii)
(C)	(p) − (ii), (q) − (iii), (r) − (i)
(D)	(p) − (iii), (q) − (ii), (r) − (i)

Column 1	**Column 2**
(p) Principal Component Analysis	(i) Discriminative Model
(q) Naïve Bayes Classification	(ii) Dimensionality Reduction
(r) Logistic Regression	(iii) Generative Model

Q.19	Euclidean distance based k-means clustering algorithm was run on a dataset of 100 points with $k = 3$. If the points $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$ are both part of cluster 3, then which **ONE** of the following points is necessarily also part of cluster 3?
(A)	$\begin{bmatrix}0\\0\end{bmatrix}$
(B)	$\begin{bmatrix}0\\2\end{bmatrix}$
(C)	$\begin{bmatrix}2\\0\end{bmatrix}$
(D)	$\begin{bmatrix}0\\1\end{bmatrix}$

Q.20	Given a dataset with K binary-valued attributes (where $K > 2$) for a two-class classification task, the number of parameters to be estimated for learning a naïve Bayes classifier is
(A)	$2^K + 1$
(B)	$2K + 1$
(C)	$2^{K+1} + 1$
(D)	$K^2 + 1$

Q.21	Consider performing uniform hashing on an open address hash table with load factor $\alpha = \frac{n}{m} < 1$, where n elements are stored in the table with m slots. The expected number of probes in an unsuccessful search is at most $\frac{1}{1-\alpha}$. Inserting an element in this hash table requires at most _______ probes, on average.
(A)	$\ln\left(\frac{1}{1-\alpha}\right)$
(B)	$\frac{1}{1-\alpha}$
(C)	$1 + \frac{\alpha}{2}$
(D)	$\frac{1}{1+\alpha}$

Q.22	For any binary classification dataset, let $S_B \in \mathbb{R}^{d\times d}$ and $S_W \in \mathbb{R}^{d\times d}$ be the between-class and within-class scatter (covariance) matrices, respectively. The Fisher linear discriminant is defined by $u^* \in \mathbb{R}^d$, that maximizes $$J(u) = \frac{u^T S_B u}{u^T S_W u}$$ If $\lambda = J(u^*)$, S_W is non-singular and $S_B \neq 0$, then (u^*, λ) must satisfy which **ONE** of the following equations? **Note**: $\mathbb{R}$ denotes the set of real numbers.
(A)	$S_W^{-1} S_B u^* = \lambda u^*$
(B)	$S_W u^* = \lambda S_B u^*$
(C)	$S_B S_W u^* = \lambda u^*$
(D)	$u^{*T} u^* = \lambda^2$

Q.23	Let h_1 and h_2 be two admissible heuristics used in A^* search. Which **ONE** of the following expressions is always an admissible heuristic?
(A)	$h_1 + h_2$
(B)	$h_1 \times h_2$
(C)	$h_1/h_2, (h_2 \neq 0)$
(D)	$\|h_1 - h_2\|$

Q.24	Consider five random variables $U, V, W, X,$ and Y whose joint distribution satisfies: $$P(U,V,W,X,Y) = P(U)P(V)P(W\|U,V)P(X\|W)P(Y\|W)$$ Which **ONE** of the following statements is **FALSE**?
(A)	Y is conditionally independent of V given W
(B)	X is conditionally independent of U given W
(C)	U and V are conditionally independent given W
(D)	Y and X are conditionally independent given W

Q.25	Consider the following statement: In adversarial search, $\alpha-\beta$ pruning can be applied to game trees of any depth where α is the __(m)__ value choice we have formed so far at any choice point along the path for the MAX player and β is the __(n)__ value choice we have formed so far at any choice point along the path for the MIN player. Which **ONE** of the following choices of **(m)** and **(n)** makes the above statement valid?
(A)	**(m)** = highest, **(n)** = highest
(B)	**(m)** = lowest, **(n)** = highest
(C)	**(m)** = highest, **(n)** = lowest
(D)	**(m)** = lowest, **(n)** = lowest

Q.26	Consider a database that includes the following relations: Defender(*name*, *rating*, *side*, *goals*) Forward(*name*, *rating*, *assists*, *goals*) Team(*name*, *club*, *price*) Which **ONE** of the following relational algebra expressions checks that every name occurring in Team appears in either Defender or Forward, where ϕ denotes the empty set?
(A)	$\Pi_{name}(\text{Team}) \setminus \left(\Pi_{name}(\text{Defender}) \cap \Pi_{name}(\text{Forward})\right) = \phi$
(B)	$(\Pi_{name}(\text{Defender}) \cap \Pi_{name}(\text{Forward})) \setminus \Pi_{name}(\text{Team}) = \phi$
(C)	$\Pi_{name}(\text{Team}) \setminus \left(\Pi_{name}(\text{Defender}) \cup \Pi_{name}(\text{Forward})\right) = \phi$
(D)	$\left(\Pi_{name}(\text{Defender}) \cup \Pi_{name}(\text{Forward})\right) \setminus \Pi_{name}(\text{Team}) = \phi$

Q.27	Let the minimum, maximum, mean and standard deviation values for the attribute *income* of data scientists be ₹46000, ₹170000, ₹96000, and ₹21000, respectively. The z-score normalized *income* value of ₹106000 is closest to which **ONE** of the following options?
(A)	0.217
(B)	0.476
(C)	0.623
(D)	2.304

Q.28	Consider the following tree traversals on a full binary tree: (i) Preorder (ii) Inorder (iii) Postorder Which of the following traversal options **is/are** sufficient to uniquely reconstruct the full binary tree?
(A)	(i) and (ii)
(B)	(ii) and (iii)
(C)	(i) and (iii)
(D)	(ii) only

Q.29	Let x and y be two propositions. Which of the following statements **is a tautology /are tautologies**?
(A)	$(\neg x \land y) \Rightarrow (y \Rightarrow x)$
(B)	$(x \land \neg y) \Rightarrow (\neg x \Rightarrow y)$
(C)	$(\neg x \land y) \Rightarrow (\neg x \Rightarrow y)$
(D)	$(x \land \neg y) \Rightarrow (y \Rightarrow x)$

Q.30 Consider sorting the following array of integers in ascending order using an in-place Quicksort algorithm that uses the last element as the pivot.

60	70	80	90	100

The minimum number of swaps performed during this Quicksort is ______.

Q.31 Consider the following two tables named Raider and Team in a relational database maintained by a Kabaddi league. The attribute *ID* in table Team references the primary key of the Raider table, *ID*.

Raider

ID	*Name*	*Raids*	*RaidPoints*
1	Arjun	200	250
2	Ankush	190	219
3	Sunil	150	200
4	Reza	150	190
5	Pratham	175	220
6	Gopal	193	215

Team

City	*ID*	*BidPoints*
Jaipur	2	200
Patna	3	195
Hyderabad	5	175
Jaipur	1	250
Patna	4	200
Jaipur	6	200

The SQL query described below is executed on this database:

```
SELECT *
FROM Raider, Team
WHERE Raider.ID=Team.ID AND City="Jaipur" AND
RaidPoints > 200;
```

The number of rows returned by this query is ______.

Q.32	The fundamental operations in a double-ended queue D are:
	
	`insertFirst(e)` – Insert a new element `e` at the beginning of D.
	`insertLast(e)` – Insert a new element `e` at the end of D.
	`removeFirst()` – Remove and return the first element of D.
	`removeLast()` – Remove and return the last element of D.
	
	In an empty double-ended queue, the following operations are performed:
	
	`insertFirst(10)`
	
	`insertLast(32)`
	
	a ←`removeFirst()`
	
	`insertLast(28)`
	
	`insertLast(17)`
	
	a ←`removeFirst()`
	
	a ← `removeLast()`
	
	The value of **a** is ______.

Q.33	Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be the function $f(x) = \frac{1}{1+e^{-x}}$. The value of the derivative of f at x where $f(x) = 0.4$ is ______ (rounded off to **two** decimal places). **Note**: $\mathbb{R}$ denotes the set of real numbers.

Q.34	The sample average of 50 data points is 40. The updated sample average after including a new data point taking the value of 142 is _______.

Q.35	Consider the 3×3 matrix $\boldsymbol{M} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \\ 4 & 3 & 6 \end{bmatrix}$. The determinant of $(\boldsymbol{M}^2 + 12\boldsymbol{M})$ is _______.

Q.36	A fair six-sided die (with faces numbered 1, 2, 3, 4, 5, 6) is repeatedly thrown independently. What is the expected number of times the die is thrown until **two** consecutive throws of even numbers are seen?
(A)	2
(B)	4
(C)	6
(D)	8

Q.37	Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. **Note**: $\mathbb{R}$ denotes the set of real numbers. $$f(x) = \begin{cases} -x, & \text{if } x < -2 \\ ax^2 + bx + c, & \text{if } x \in [-2,2] \\ x, & \text{if } x > 2 \end{cases}$$ Which **ONE** of the following choices gives the values of a, b, c that make the function f continuous and differentiable?
(A)	$a = \frac{1}{4}, b = 0, c = 1$
(B)	$a = \frac{1}{2}, b = 0, c = 0$
(C)	$a = 0, b = 0, c = 0$
(D)	$a = 1, b = 1, c = -4$

Q.38	Consider the following Python code:
	
	`def count(child_dict, i):`
	`    if i not in child_dict.keys():`
	`         return 1`
	`    ans = 1`
	`    for j in child_dict[i]:`
	`        ans += count(child_dict, j)`
	`    return ans`
	
	`child_dict = dict()`
	`child_dict[0] = [1,2]`
	`child_dict[1] = [3,4,5]`
	`child_dict[2] = [6,7,8]`
	`print(count(child_dict,0))`
	
	Which **ONE** of the following is the output of this code?

(A)	6
(B)	1
(C)	8
(D)	9

Q.39	Consider the function **computeS**(X) whose pseudocode is given below:
	
	computeS(X)
	$S[1] \leftarrow 1$
	for i ←2 to $length(X)$
	  $S[i] \leftarrow 1$
	  if $X[i-1] \leq X[i]$
	    $S[i] \leftarrow S[i] + S[i-1]$
	  end if
	end for
	return S
	
	Which **ONE** of the following values is returned by the function **computeS**(X) for $X = [6, 3, 5, 4, 10]$?

(A)	[1, 1, 2, 3, 4]
(B)	[1, 1, 2, 3, 3]
(C)	[1, 1, 2, 1, 2]
(D)	[1, 1, 2, 1, 5]

Q.40	Let $F(n)$ denote the maximum number of comparisons made while searching for an entry in a sorted array of size n using binary search. Which **ONE** of the following options is **TRUE**?
(A)	$F(n) = F(\lfloor n/2 \rfloor) + 1$
(B)	$F(n) = F(\lfloor n/2 \rfloor) + F(\lceil n/2 \rceil)$
(C)	$F(n) = F(\lfloor n/2 \rfloor)$
(D)	$F(n) = F(n-1) + 1$

Q.41 Consider the following Python function:

```
def fun(D, s1, s2):
        if s1 < s2:
            D[s1], D[s2] = D[s2], D[s1]
            fun(D, s1+1, s2-1)
```

What does this Python function `fun()` do? Select the **ONE** appropriate option below.

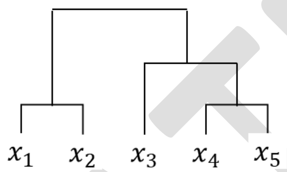
(A)	It finds the smallest element in D from index s1 to s2, both inclusive.
(B)	It performs a merge sort in-place on this list D between indices s1 and s2, both inclusive.
(C)	It reverses the list D between indices s1 and s2, both inclusive.
(D)	It swaps the elements in D at indices s1 and s2, and leaves the remaining elements unchanged.

Q.42 Consider the table below, where the $(i, j)^{th}$ element of the table is the distance between points x_i and x_j. Single linkage clustering is performed on data points, x_1, x_2, x_3, x_4, x_5.

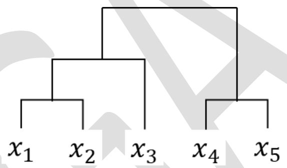
	x_1	x_2	x_3	x_4	x_5
x_1	0	1	4	3	6
x_2	1	0	3	5	3
x_3	4	3	0	2	5
x_4	3	5	2	0	1
x_5	6	3	5	1	0

Which **ONE** of the following is the correct representation of the clusters produced?

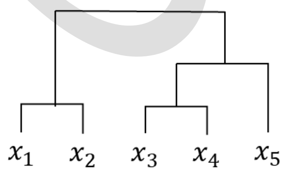
(A)



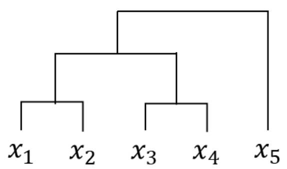
(B)

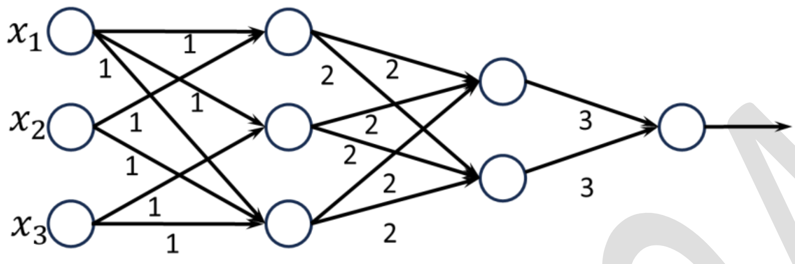
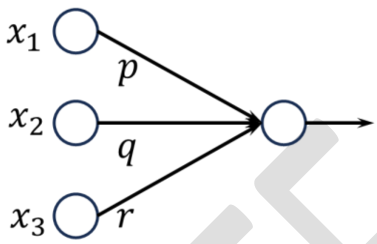


(C)



(D)



Q.43	Consider the two neural networks (NNs) shown in Figures 1 and 2, with $ReLU$ activation ($ReLU(z) = \max\{0, z\}, \forall z \in \mathbb{R}$). $\mathbb{R}$ denotes the set of real numbers. The connections and their corresponding weights are shown in the Figures. The biases at every neuron are set to 0. For what values of p, q, r in Figure 2 are the two NNs equivalent, when x_1, x_2, x_3 are positive? **Figure 1**  **Figure 2** 
(A)	$p = 36, q = 24, r = 24$
(B)	$p = 24, q = 24, r = 36$
(C)	$p = 18, q = 36, r = 24$
(D)	$p = 36, q = 36, r = 36$

Q.44	Consider a state space where the start state is number 1. The successor function for the state numbered n returns two states numbered $n+1$ and $n+2$. Assume that the states in the unexpanded state list are expanded in the ascending order of numbers and the previously expanded states are not added to the unexpanded state list. Which **ONE** of the following statements about breadth-first search (BFS) and depth-first search (DFS) is true, when reaching the goal state number 6?
(A)	BFS expands more states than DFS.
(B)	DFS expands more states than BFS.
(C)	Both BFS and DFS expand equal number of states.
(D)	Both BFS and DFS do not reach the goal state number 6.

Q.45	Consider the following sorting algorithms: (i) Bubble sort (ii) Insertion sort (iii) Selection sort Which **ONE** among the following choices of sorting algorithms sorts the numbers in the array [4, 3, 2, 1, 5] in increasing order after **exactly two** passes over the array?
(A)	(i) only
(B)	(iii) only
(C)	(i) and (iii) only
(D)	(ii) and (iii) only

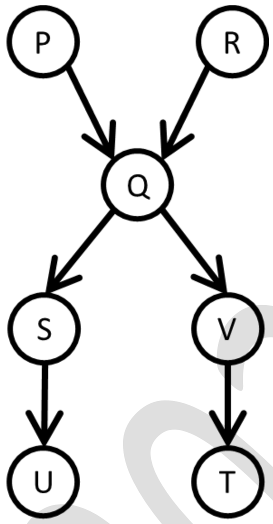
Q.46	Given the relational schema $R = (U, V, W, X, Y, Z)$ and the set of functional dependencies: $$\{U \to V, U \to W, WX \to Y, WX \to Z, V \to X\}$$ Which of the following functional dependencies can be derived from the above set?
(A)	$VW \to YZ$
(B)	$WX \to YZ$
(C)	$VW \to U$
(D)	$VW \to Y$

Q.47	Select all choices that are subspaces of $\mathbb{R}^3$. **Note**: $\mathbb{R}$ denotes the set of real numbers.
(A)	$\left\{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \ \mathbf{x} = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \alpha, \beta \in \mathbb{R}\right\}$
(B)	$\left\{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \ \mathbf{x} = \alpha^2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \beta^2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \alpha, \beta \in \mathbb{R}\right\}$
(C)	$\left\{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \ 5x_1 + 2x_3 = 0, 4x_1 - 2x_2 + 3x_3 = 0\right\}$
(D)	$\left\{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \ 5x_1 + 2x_3 + 4 = 0\right\}$

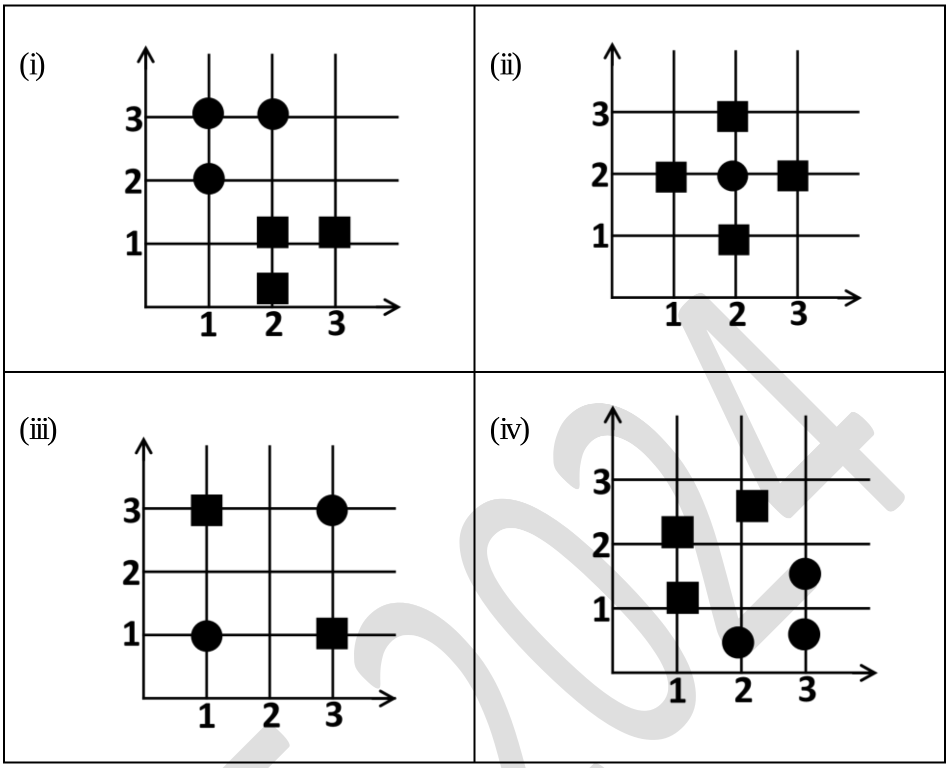
Q.48	Which of the following statements is/are **TRUE**? **Note**: $\mathbb{R}$ denotes the set of real numbers.
(A)	There exist $\boldsymbol{M} \in \mathbb{R}^{3\times 3}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions.
(B)	There exist $\boldsymbol{M} \in \mathbb{R}^{3\times 3}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has no solutions and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions.
(C)	There exist $\boldsymbol{M} \in \mathbb{R}^{2\times 3}, \boldsymbol{p} \in \mathbb{R}^2$, and $\boldsymbol{q} \in \mathbb{R}^2$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions.
(D)	There exist $\boldsymbol{M} \in \mathbb{R}^{3\times 2}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has no solutions.

Q.49	Let $\mathbb{R}$ be the set of real numbers, U be a subspace of $\mathbb{R}^3$ and $\boldsymbol{M} \in \mathbb{R}^{3\times 3}$ be the matrix corresponding to the projection on to the subspace U. Which of the following statements is/are **TRUE**?
(A)	If U is a 1-dimensional subspace of $\mathbb{R}^3$, then the null space of $\boldsymbol{M}$ is a 1-dimensional subspace.
(B)	If U is a 2-dimensional subspace of $\mathbb{R}^3$, then the null space of $\boldsymbol{M}$ is a 1-dimensional subspace.
(C)	$\boldsymbol{M}^2 = \boldsymbol{M}$
(D)	$\boldsymbol{M}^3 = \boldsymbol{M}$

Q.50	Consider the function $f: \mathbb{R} \to \mathbb{R}$ where $\mathbb{R}$ is the set of all real numbers. $$f(x) = \frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2} + 1$$ Which of the following statements is/are **TRUE**?
(A)	$x = 0$ is a local maximum of f
(B)	$x = 3$ is a local minimum of f
(C)	$x = -1$ is a local maximum of f
(D)	$x = 0$ is a local minimum of f

Q.51	Consider the directed acyclic graph (DAG) below:  Which of the following is/are valid vertex orderings that can be obtained from a topological sort of the DAG?
(A)	P Q R S T U V
(B)	P R Q V S U T
(C)	P Q R S V U T
(D)	P R Q S V T U

Q.52	Let $H, I, L,$ and N represent height, number of internal nodes, number of leaf nodes, and the total number of nodes respectively in a rooted binary tree. Which of the following statements is/are always **TRUE**?
(A)	$L \leq I + 1$
(B)	$H + 1 \leq N \leq 2^{H+1} - 1$
(C)	$H \leq I \leq 2^{H} - 1$
(D)	$H \leq L \leq 2^{H-1}$

Q.53	Consider the following figures representing datasets consisting of two-dimensional features with two classes denoted by circles and squares.  Which of the following is/are **TRUE**?
(A)	(i) is linearly separable.
(B)	(ii) is linearly separable.
(C)	(iii) is linearly separable.
(D)	(iv) is linearly separable.

Q.54	Let *game(ball, rugby)* be true if the ball is used in rugby and false otherwise. Let *shape(ball, round)* be true if the ball is round and false otherwise. Consider the following logical sentences: $s1: \forall ball \neg game(ball, rugby) \Longrightarrow shape(ball, round)$ $s2: \forall ball \neg shape(ball, round) \Longrightarrow game(ball, rugby)$ $s3: \forall ball\ game(ball, rugby) \Longrightarrow \neg shape(ball, round)$ $s4: \forall ball\ shape(ball, round) \Longrightarrow \neg game(ball, rugby)$ Which of the following choices is/are logical representations of the assertion, "All balls are round except balls used in rugby"?
(A)	$s1 \wedge s3$
(B)	$s1 \wedge s2$
(C)	$s2 \wedge s3$
(D)	$s3 \wedge s4$

Q.55	An OTT company is maintaining a large disk-based relational database of different movies with the following schema:
	
	`Movie(ID, CustomerRating)`
	`Genre(ID, Name)`
	`Movie_Genre(MovieID, GenreID)`
	
	Consider the following SQL query on the relation database above:
	
	`SELECT *`
	`FROM Movie, Genre, Movie_Genre`
	`WHERE`
	`    Movie.CustomerRating > 3.4 AND`
	`    Genre.Name = “Comedy” AND`
	`    Movie_Genre.MovieID = Movie.ID AND`
	`    Movie_Genre.GenreID = Genre.ID;`
	
	This SQL query can be sped up using which of the following indexing options?

A	B^+ tree on all the attributes.
B	Hash index on `Genre.Name` and B^+ tree on the remaining attributes.
C	Hash index on `Movie.CustomerRating` and B^+ tree on the remaining attributes.
D	Hash index on all the attributes.

Q.56	Let X be a random variable uniformly distributed in the interval [1, 3] and Y be a random variable uniformly distributed in the interval [2, 4]. If X and Y are independent of each other, the probability $P(X \geq Y)$ is ______ (rounded off to **three** decimal places).

Q.57	Let X be a random variable exponentially distributed with parameter $\lambda > 0$. The probability density function of X is given by: $$f_X(x) = \begin{cases} \lambda\, e^{-\lambda x}, & x \geq 0 \\ 0, & otherwise \end{cases}$$ If $5E(X) = Var(X)$, where $E(X)$ and $Var(X)$ indicate the expectation and variance of X, respectively, the value of λ is ______ (rounded off to **one** decimal place).

Q.58	Consider two events T and S. Let $\bar{T}$ denote the complement of the event T. The probability associated with different events are given as follows: $$P(\bar{T}) = 0.6, \qquad P(S\|T) = 0.3, \qquad P(S\|\bar{T}) = 0.6$$ Then, $P(T\|S)$ is ______ (rounded off to **two** decimal places).

Q.59	Consider a joint probability density function of two random variables X and Y $$f_{X,Y}(x,y) = \begin{cases} 2xy, & 0 < x < 2, \quad 0 < y < x \\ 0, & \text{otherwise} \end{cases}$$ Then, $E[Y\|X = 1.5]$ is ______.

Q.60	Evaluate the following limit: $$\lim_{x \to 0} \frac{\ln((x^2+1)\cos x)}{x^2} = \_\_\_\_\_\_.$$

Q.61	Let $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$, and let $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ be the singular values of the matrix $\boldsymbol{M} = \boldsymbol{u}\boldsymbol{u}^{\boldsymbol{T}}$ (where $\boldsymbol{u}^{\boldsymbol{T}}$ is the transpose of $\boldsymbol{u}$). The value of $\sum_{i=1}^{5} \sigma_i$ is _______.

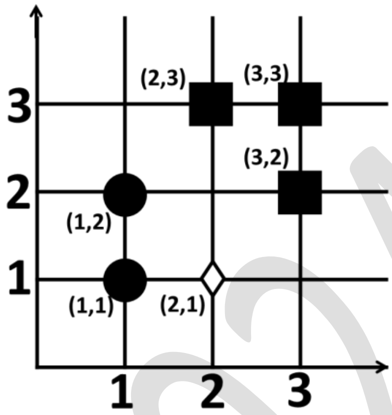
Q.62 Details of ten international cricket games between two teams "Green" and "Blue" are given in Table C. This table consists of matches played on different pitches, across formats along with their winners. The attribute *Pitch* can take one of two values: spin-friendly (represented as S) or pace-friendly (represented as F). The attribute *Format* can take one of two values: one-day match (represented as O) or test match (represented as T).

A cricket organization would like to use the information given in Table C to develop a decision-tree model to predict outcomes of future games between these two teams.

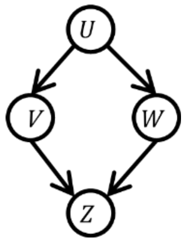
To develop such a model, the computed InformationGain(C, *Pitch*) with respect to the Target is _______ (rounded off to **two** decimal places).

Table C

Match Number	Pitch	Format	Winner (Target)
1	S	T	Green
2	S	T	Blue
3	F	O	Blue
4	S	O	Blue
5	F	T	Green
6	F	O	Blue
7	S	O	Green
8	F	T	Blue
9	F	O	Blue
10	S	O	Green

Q.63	Given the two-dimensional dataset consisting of 5 data points from two classes (circles and squares) and assume that the Euclidean distance is used to measure the distance between two points. The minimum odd value of k in k-nearest neighbor algorithm for which the diamond (◊) shaped data point is assigned the label square is ______. 

Q.64	Given the following Bayesian Network consisting of four Bernoulli random variables and the associated conditional probability tables:



	$P(\cdot)$
$U=0$	0.5
$U=1$	0.5

	$P(V=0\mid\cdot)$	$P(V=1\mid\cdot)$
$U=0$	0.5	0.5
$U=1$	0.5	0.5

	$P(W=0\mid\cdot)$	$P(W=1\mid\cdot)$
$U=0$	1	0
$U=1$	0	1

		$P(Z=0\mid\cdot)$	$P(Z=1\mid\cdot)$
$V=0$	$W=0$	0.5	0.5
$V=0$	$W=1$	1	0
$V=1$	$W=0$	1	0
$V=1$	$W=1$	0.5	0.5

The value of $P(U=1, V=1, W=1, Z=1) =$ _______ (rounded off to **three** decimal places).

Q.65	Two fair coins are tossed independently. X is a random variable that takes a value of 1 if both tosses are heads and 0 otherwise. Y is a random variable that takes a value of 1 if at least one of the tosses is heads and 0 otherwise. The value of the covariance of X and Y is _______ (rounded off to **three** decimal places).