General Aptitude

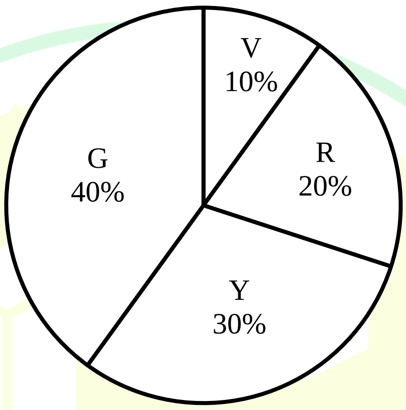
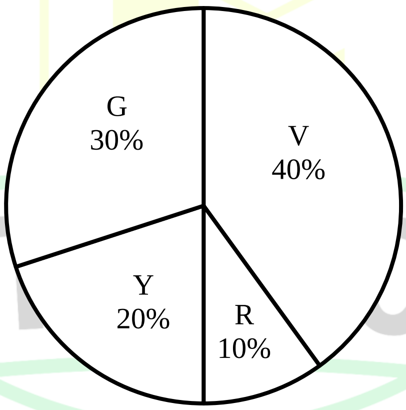
Q.1 – Q.5 Carry ONE mark Each

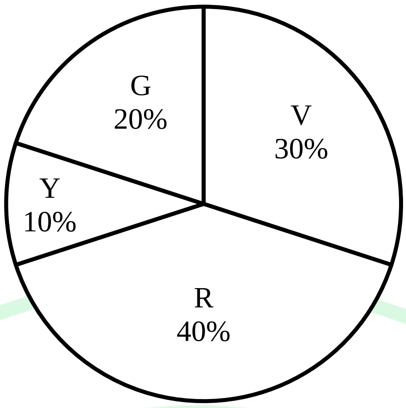
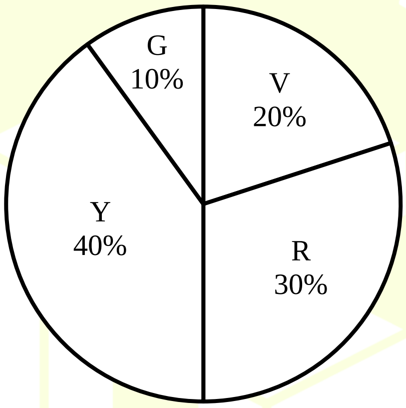
Q.1	Even though I had planned to go skiing with my friends, I had to ___________ at the last moment because of an injury. Select the most appropriate option to complete the above sentence.
(A)	back up
(B)	back of
(C)	back on
(D)	back out
Q.2	The President, along with the Council of Ministers, ____________ to visit India next week. Select the most appropriate option to complete the above sentence.
(A)	wish
(B)	wishes
(C)	will wish
(D)	is wishing

Q.3	An electricity utility company charges ₹ 7 per kWh (kilo watt-hour). If a 40-watt desk light is left on for 10 hours each night for 180 days, what would be the cost of energy consumption? If the desk light is on for 2 more hours each night for the 180 days, what would be the percentage-increase in the cost of energy consumption?
(A)	₹ 604.8; 10%
(B)	₹ 504; 20%
(C)	₹ 604.8; 12%
(D)	₹ 720; 15%

Q.4	In the context of the given figure, which one of the following options correctly represents the entries in the blocks labelled (i), (ii), (iii), and (iv), respectively?
(A)	Q, M, 12, and 8
(B)	K, L, 10 and 14
(C)	I, J, 10, and 8
(D)	L, K, 12 and 8

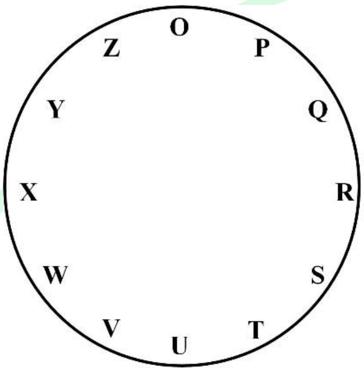
N	U	F	(i)
21	14	9	6
H	L	(ii)	O
12	(iv)	15	(iii)

Q.5	A bag contains Violet (V), Yellow (Y), Red (R), and Green (G) balls. On counting them, the following results are obtained: (i) The sum of Yellow balls and twice the number of Violet balls is 50. (ii) The sum of Violet and Green balls is 50. (iii) The sum of Yellow and Red balls is 50. (iv) The sum of Violet and twice the number of Red balls is 50. Which one of the following Pie charts correctly represents the balls in the bag?
(A)	
(B)	

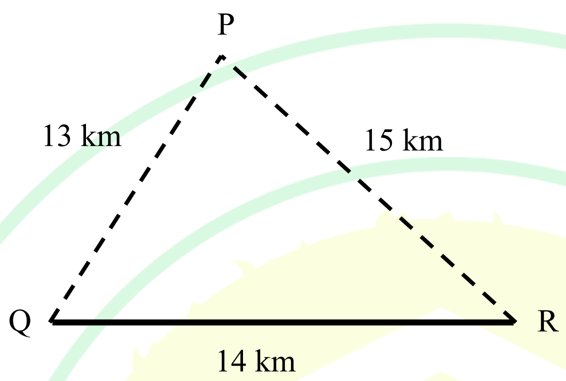
(C)	G 20% V 30% Y 10% R 40%
(D)	G 10% V 20% Y 40% R 30%

Q.6 – Q.10 Carry TWO marks Each

Q.6	“His life was divided between the books, his friends, and long walks. A solitary man, he worked at all hours without much method, and probably courted his fatal illness in this way. To his own name there is not much to show; but such was his liberality that he was continually helping others, and fruits of his erudition are widely scattered, and have gone to increase many a comparative stranger’s reputation.” (From E.V. Lucas’s “A Funeral”) Based only on the information provided in the above passage, which one of the following statements is true?
(A)	The solitary man described in the passage is dead.
(B)	Strangers helped create a grand reputation for the solitary man described in the passage.
(C)	The solitary man described in the passage found joy in scattering fruits.
(D)	The solitary man worked in a court where he fell ill.

Q.7	For the clock shown in the figure, if $O^* =$ O Q S Z P R T, and $X^* =$ X Z P W Y O Q, then which one among the given options is most appropriate for P^* ?
	
(A)	P U W R T V X
(B)	P R T O Q S U
(C)	P T V Q S U W
(D)	P S U P R T V

Q.8	Consider a five-digit number $PQRST$ that has distinct digits $P, Q, R, S,$ and T, and satisfies the following conditions: $$P < Q$$ $$S > P > T$$ $$R < T$$ If integers 1 through 5 are used to construct such a number, the value of P is:
(A)	1
(B)	2
(C)	3
(D)	4
Q.9	A business person buys potatoes of two different varieties P and Q, mixes them in a certain ratio and sells them at ₹ 192 per kg. The cost of the variety P is ₹ 800 for 5 kg. The cost of the variety Q is ₹ 800 for 4 kg. If the person gets 8% profit, what is the P:Q ratio (by weight)?
(A)	5:4
(B)	3:4
(C)	3:2
(D)	1:1

Q.10	Three villages P, Q, and R are located in such a way that the distance PQ = 13 km, QR = 14 km, and RP = 15 km, as shown in the figure. A straight road joins Q and R. It is proposed to connect P to this road QR by constructing another road. What is the minimum possible length (in km) of this connecting road? Note: The figure shown is representative.
	
(A)	10.5
(B)	11.0
(C)	12.0
(D)	12.5

Q.11 – Q.35 Carry ONE mark Each

Q.11	Let $f:[0,\infty)\to[0,\infty)$ be a differentiable function with $f(x)>0$ for all $x>0$, and $f(0)=0$. Further, f satisfies $$(f(x))^2=\int_0^x\left(\left(f(t)\right)^2+f(t)\right)dt\ ,\ \ x>0.$$ Then which one of the following options is correct?
(A)	$0<f(2)\leq 1$
(B)	$1<f(2)\leq 2$
(C)	$2<f(2)\leq 3$
(D)	$3<f(2)\leq 4$
Q.12	Among the following four statements about countability and uncountability of different sets, which is the correct statement?
(A)	The set $\bigcup_{n=0}^{\infty}\left\{x\in\mathbb{R}: x=\sum_{i=0}^{n}10^i a_i\text{, where } a_i\in\{1,2\}\text{ for } i=0,1,2,\dots,n\right\}$ is uncountable
(B)	The set $\left\{x\in(0,1): x=\sum_{n=1}^{\infty}\frac{a_n}{10^n}\text{, where } a_n=1\text{ or }2\text{ for each } n\in\mathbb{N}\right\}$ is uncountable
(C)	There exists an uncountable set whose elements are pairwise disjoint open intervals in $\mathbb{R}$
(D)	The set of all intervals with rational end points is uncountable

Q.13	Let $S = \{(x,y,z) \in \mathbb{R}^3 \setminus \{(0,0,0)\} : z = -(x+y)\}$. Denote $S^\perp = \{(p,q,r) \in \mathbb{R}^3 : px + qy + rz = 0 \text{ for all } (x,y,z) \in S\}$. Then which one of the following options is correct?
(A)	$S^\perp$ is not a subspace of $\mathbb{R}^3$
(B)	$S^\perp = \{(0,0,0)\}$
(C)	$\dim(S^\perp) = 1$
(D)	$\dim(S^\perp) = 2$
Q.14	Let X be a random variable having the Poisson distribution with mean $\log_e 2$. Then $E\left(e^{(\log_e 3)X}\right)$ equals
(A)	1
(B)	2
(C)	3
(D)	4

Q.15	Let (X_1, X_2, X_3) follow the multinomial distribution with the number of trials being 100 and the probability vector $\left(\frac{3}{10}, \frac{1}{10}, \frac{3}{5}\right)$. Then $\mathrm{E}(\mathrm{X}_2 \mid X_3 = 40)$ equals
(A)	25
(B)	15
(C)	30
(D)	45
Q.16	Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables with the common probability density function $$f(x) = \frac{1}{\pi(1 + x^2)}, \quad -\infty < x < \infty .$$ Define $Y_n = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(X_n)$ for $n = 1, 2, \ldots$. Then which one of the following options is correct?
(A)	$\frac{1}{n}\sum_{i=1}^{n} Y_i \xrightarrow{P} \frac{1}{2}$ as $n \to \infty$
(B)	$\frac{1}{n}\sum_{i=1}^{n} Y_i \xrightarrow{P} 0$ as $n \to \infty$
(C)	$\frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{P} 0$ as $n \to \infty$
(D)	$\frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{P} \frac{1}{2}$ as $n \to \infty$

Q.17	Let $\{N(t): t \geq 0\}$ be a homogenous Poisson process with the intensity/rate $\lambda = 2$. Let $$X = N(6) - N(1)$$ $$Y = N(5) - N(3)$$ $$W = N(6) - N(5)$$ $$Z = N(3) - N(1).$$ Then which one of the following options is correct?
(A)	$\mathrm{Cov}(W, Z) = 2$
(B)	$Y + Z \sim \mathrm{Poisson}(10)$
(C)	$\Pr(Y = Z) = 1$
(D)	$\mathrm{Cov}(X, Y) = 4$
Q.18	Let T be a complete and sufficient statistic for a family $\mathcal{P}$ of distributions and let U be a sufficient statistic for $\mathcal{P}$. If $P_f(T \geq 0) = 1$ for all $f \in \mathcal{P}$, then which one of the following options is NOT necessarily correct?
(A)	T^2 is a complete statistic for $\mathcal{P}$
(B)	T^2 is a minimal sufficient statistic for $\mathcal{P}$
(C)	T is a function of U
(D)	U is a function of T

Q.19	Let X_1, X_2 be a random sample from $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$. Consider testing $H_0: \theta = 0$ against $H_1: \theta \neq 0$. Let $\phi(X_1, X_2)$ be the likelihood ratio test of size 0.05 for testing H_0 against H_1. Then which one of the following options is correct?
(A)	$\phi(X_1, X_2)$ is a uniformly most powerful test of size 0.05
(B)	$\mathrm{E}_\theta\big(\phi(X_1, X_2)\big) \geq 0.05 \ \ \forall\, \theta \in \mathbb{R}$
(C)	There exists a uniformly most powerful test of size 0.05
(D)	$\mathrm{E}_{\theta=0}\big(X_1 \phi(X_1, X_2)\big) = 0.05$
Q.20	Let a random variable X follow a distribution with density $f \in \{f_0, f_1\}$, where $$f_0(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad f_1(x) = \begin{cases} 1 & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$ Let ϕ be a most powerful test of level 0.05 for testing $H_0: f = f_0$ against $H_1: f = f_1$ based on X. Then which one of the following options is necessarily correct?
(A)	$\mathrm{E}_{f_0}\big(\phi(X)\big) = 0.05$
(B)	$\mathrm{E}_{f_1}\big(\phi(X)\big) = 1$
(C)	$P_f(\phi(X) = 1) = P_f(X > 1), \ \ \forall\, f \in \{f_0, f_1\}$
(D)	$P_{f_1}(\phi(X) = 1) < 1$

Q.21	Let X be a random variable having probability density function $f \in \{f_0, f_1\}$. Let ϕ be a most powerful test of level 0.05 for testing $H_0: f = f_0$ against $H_1: f = f_1$ based on X. Then which one of the following options is NOT necessarily correct?
(A)	ϕ is the unique most powerful test of level 0.05
(B)	$\mathrm{E}_{f_1}\big(\phi(X)\big) \geq 0.05$
(C)	$\mathrm{E}_{f_0}\big(\phi(X)\big) \leq 0.05$
(D)	For some constant $c \geq 0$, $P_f\big(f_1(X) > c f_0(X)\big) \leq P_f(\phi(X) = 1)$, $\forall\, f \in \{f_0, f_1\}$
Q.22	Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables with common distribution function F, and let F_n be the empirical distribution function based on $\{X_1, X_2, \ldots, X_n\}$. Then, for each fixed $x \in (-\infty, \infty)$, which one of the following options is correct?
(A)	$\sqrt{n}\big(F_n(x) - F(x)\big) \xrightarrow{P} 0$ as $n \to \infty$
(B)	$\dfrac{n\big(F_n(x) - F(x)\big)}{\sqrt{F(x)\big(1 - F(x)\big)}} \xrightarrow{d} Z$ as $n \to \infty$, where $Z \sim N(0,1)$
(C)	$F_n(x) \xrightarrow{a.s.} F(x)$ as $n \to \infty$
(D)	$\lim_{n\to\infty} n\,\mathrm{Var}\big(F_n(x)\big) = 0$

Q.23	Let $(X, Y)^T$ follow a bivariate normal distribution with $\mathrm{E}(X) = 3, \mathrm{E}(Y) = 4$, $\mathrm{Var}(X) = 25, \mathrm{Var}(Y) = 100$, and $\mathrm{Cov}(X, Y) = 50\rho$, where $\rho \in (-1,1)$. If $\mathrm{E}(Y \mid X = 5) = 4.32$, then ρ equals
(A)	0.08
(B)	0.8
(C)	0.32
(D)	0.5
Q.24	For a given data $(x_i, y_i), i = 1, 2, \dots, n$, with $\sum_{i=1}^{n} x_i^2 > 0$, let $\hat{\beta}$ satisfy $$\sum_{i=1}^{n} \left(y_i - \hat{\beta} x_i\right)^2 = \inf_{\beta \in \mathbb{R}} \sum_{i=1}^{n} (y_i - \beta x_i)^2.$$ Further, let $v_j = y_j - x_j$ and $u_j = 2x_j$, for $j = 1,2, \dots, n$, and let $\hat{\gamma}$ satisfy $$\sum_{i=1}^{n} (v_i - \hat{\gamma} u_i)^2 = \inf_{\gamma \in \mathbb{R}} \sum_{i=1}^{n} (v_i - \gamma u_i)^2 .$$ If $\hat{\beta} = 10$, then the value of $\hat{\gamma}$ is
(A)	4.5
(B)	5
(C)	10
(D)	9

Q.25	Let $$I = \pi^2 \int_0^1 \int_0^1 y^2 \cos \pi(1 + xy)\, dx\, dy\,.$$ The value of I is equal to ____________ (*answer in integer*).
Q.26	Let $P = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$ and $Q = P^3 - 2P^2 - 4P + 13I_2$, where I_2 denotes the identity matrix of order 2. Then the determinant of Q is equal to _____________ (*answer in integer*).
Q.27	Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map defined by $$T(x_1, x_2, x_3) = (3x_1 + 5x_2 + x_3,\ \ x_3,\ \ 2x_1 + 2x_3).$$ Then the rank of T is equal to _____________ (*answer in integer*).
Q.28	Let X be a random variable with distribution function F, such that $$\lim_{h \to 0^-} F(3 + h) = \frac{1}{4} \text{ and } F(3) = \frac{3}{4}.$$ Then $16\,\Pr(X = 3)$ equals _____________ (*answer in integer*).

Q.29	Let $X \sim Bin\left(2, \frac{1}{3}\right)$. Then $18\,\mathrm{E}(X^2)$ equals ____________ (*answer in integer*).
Q.30	Let X follow a 10-dimensional multivariate normal distribution with zero mean vector and identity covariance matrix. Define $Y = \log_e \sqrt{X^T X}$ and let $M_Y(t)$ denote the moment generating function of Y at t, $t > -10$. Then $M_Y(2)$ equals ____________ (*answer in integer*).
Q.31	Let $\{W(t): t \geq 0\}$ be a standard Brownian motion. Then $\mathrm{E}\left(\left(W(2) + W(3)\right)^2\right)$ equals ____________ (*answer in integer*).
Q.32	Let $x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 1$ and $x_5 = 0$ be observed values of a random sample of size 5 from $Bin(1, \theta)$ distribution, where $\theta \in (0, 0.7]$. Then the maximum likelihood estimate of θ based on the above sample is ____________ (*rounded off to two decimal places*).
Q.33	Let $X_1, \ldots, X_5$ be a random sample from $N(\theta, 6)$, where $\theta \in \mathbb{R}$, and let $c(\theta)$ be the Cramer-Rao lower bound for the variances of unbiased estimators of θ based on the above sample. Then $15 \inf_{\theta \in \mathbb{R}} c(\theta)$ equals ____________ (*answer in integer*).

Q.34	Let $(1,3),(2,4),(7,8)$ be three independent observations. Then the sample Spearman rank correlation coefficient based on the above observations is ____________ (*rounded off to two decimal places*).
Q.35	Consider the multi-linear regression model $$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \epsilon_i \ , i = 1, 2, \dots, 25 \ ,$$ where $\beta_i, i = 0, 1, 2, 3, 4$, are unknown parameters, the errors $\epsilon_i's$ are i.i.d. random variables having $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is unknown. Suppose that the value of the coefficient of determination R^2 is obtained as $\frac{5}{6}$. Then the value of adjusted R^2 is __________ (*rounded off to two decimal places*).

Q.36 – Q.65 Carry TWO marks Each

Q.36	Let $\mathcal{F} = \{f : [a,b] \to \mathbb{R} \mid f$ is continuous on [a, b] and differentiable on $(a,b)\}$. Which one of the following options is correct?
(A)	There exists a non-constant $f \in \mathcal{F}$ such that $\lvert f(x) - f(y)\rvert \le \lvert x - y\rvert^2$ for all $x, y \in [a,b]$
(B)	If $f \in \mathcal{F}$ and $x_0 \in (a,b)$, then there exist distinct $x_1, x_2 \in [a,b]$ such that $\frac{f(x_1)-f(x_2)}{x_1-x_2} = f'(x_0)$
(C)	Let $f \in \mathcal{F}$ and $f'(x) \ge 0$ for all $x \in (a,b)$. If f' is zero only at two distinct points, then f is strictly increasing
(D)	Let $f \in \mathcal{F}$. If $f'(x_1) < c < f'(x_2)$ for some $x_1, x_2 \in (a,b)$, then there may NOT exist an $x_0 \in (x_1, x_2)$ such that $f'(x_0) = c$
Q.37	Let $U = \{(x,y) \in \mathbb{R}^2 : x + y \le 2\}$. Define $f : U \to \mathbb{R}$ by $$f(x,y) = (x-1)^4 + (y-2)^4.$$ The minimum value of f over U is
(A)	0
(B)	$\frac{1}{16}$
(C)	$\frac{17}{81}$
(D)	$\frac{1}{8}$

Q.38	Let $P = (a_{ij})$ be a 10×10 matrix with $$a_{ij} = \begin{cases} -\frac{1}{10} & \text{if } i \neq j \\ \frac{9}{10} & \text{if } i = j. \end{cases}$$ Then $\text{rank}(P)$ equals
(A)	10
(B)	9
(C)	1
(D)	8
Q.39	Let X be a random variable with the distribution function $$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \alpha(1 + 2x^2) & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1, \end{cases}$$ where α is a real constant. If the median of X is $\frac{1}{\sqrt{2}}$, then the value of α equals
(A)	$\frac{1}{2}$
(B)	$\frac{1}{3}$
(C)	$\frac{1}{4}$
(D)	$\frac{1}{6}$

Q.40	Let X be a continuous random variable with probability density function $$f(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log_e x-\mu}{\sigma}\right)^2}, \qquad x > 0,$$ where $\mu \in \mathbb{R}, \sigma > 0$. If $\log_e\left(\frac{\mathrm{E}(X^2)}{(\mathrm{E}(X))^2}\right) = 4$, then $\mathrm{Var}(\log_e X)$ equals
(A)	2
(B)	4
(C)	16
(D)	64
Q.41	Let X and Y be discrete random variables with joint probability mass function $$p_{X,Y}(m,n) = \frac{\lambda^n e^{-\lambda}}{2^n\, m!\,(n-m)!}, \quad m = 0, \dots, n, \quad \text{and} \quad n = 0, 1, 2, \dots,$$ where λ is a fixed positive real number. Then which one of the following options is correct?
(A)	The marginal distribution of X is Poisson with mean λ
(B)	The marginal distribution of Y is Poisson with mean 2λ
(C)	The conditional distribution of X given $Y = 3$ is $Bin\left(3, \frac{1}{2}\right)$
(D)	$\mathrm{E}(Y \mid X = 2) = \frac{\lambda}{2}$

Q.42	Let $X_1, \ldots, X_n, n \geq 2$, be a random sample from a $N(-\theta, \theta)$ distribution, where $\theta > 0$ is an unknown parameter. Then which one of the following options is correct?
(A)	$\sum_{i=1}^{n} X_i$ is a minimal sufficient statistic
(B)	$\sum_{i=1}^{n} {X_i}^2$ is a minimal sufficient statistic
(C)	$\left(\frac{1}{n}\sum_{i=1}^{n} X_i, \frac{1}{n-1}\sum_{j=1}^{n}\left(X_j - \frac{1}{n}\sum_{i=1}^{n} X_i\right)^2\right)$ is a complete statistic
(D)	$-\frac{1}{n}\sum_{i=1}^{n} X_i$ is a uniformly minimum variance unbiased estimator of θ

Q.43	Let X_1, X_2 be a random sample from a distribution having probability density function $$f_\theta(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$ where $\theta \in (0, \infty)$ is an unknown parameter. For testing $H_0: \theta \le 1$ against $H_1: \theta > 1$, consider the test $$\phi(X_1, X_2) = \begin{cases} 1 & \text{if } X_1 > 1 \\ 0 & \text{otherwise.} \end{cases}$$ Then which one of the following tests has the same power function as ϕ?
(A)	$\phi_1(X_1, X_2) = \begin{cases} \frac{X_1+X_2-1}{X_1+X_2} & \text{if } X_1 + X_2 > 1 \\ 0 & \text{otherwise} \end{cases}$
(B)	$\phi_2(X_1, X_2) = \begin{cases} \frac{2X_1+2X_2-1}{2(X_1+X_2)} & \text{if } X_1 + X_2 > 1 \\ 0 & \text{otherwise} \end{cases}$
(C)	$\phi_3(X_1, X_2) = \begin{cases} \frac{3X_1+3X_2-1}{3(X_1+X_2)} & \text{if } X_1 + X_2 > 1 \\ 0 & \text{otherwise} \end{cases}$
(D)	$\phi_4(X_1, X_2) = \begin{cases} \frac{4X_1+4X_2-1}{4(X_1+X_2)} & \text{if } X_1 + X_2 > 1 \\ 0 & \text{otherwise} \end{cases}$

Q.44	Let X, Y_1, Y_2 be independent random variables such that X has the probability density function $$f(x) = \begin{cases} 2e^{-2x} & \text{if } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$ and Y_1 and Y_2 are identically distributed with probability density function $$g(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$ For $i = 1, 2$, let R_i denote the rank of Y_i among X, Y_1, Y_2. Then $\mathrm{E}(R_1 + R_2)$ equals
(A)	$\frac{13}{3}$
(B)	$\frac{22}{5}$
(C)	$\frac{21}{5}$
(D)	$\frac{9}{2}$

Q.45	Let $X_1, X_2, \dots, X_5$ be i.i.d. random vectors following the bivariate normal distribution with zero mean vector and identity covariance matrix. Define 5×2 matrix X as $X = (X_1, X_2, \dots, X_5)^T$. Further, let $W = \left(W_{ij}\right) = X^T X$, and $Z = W_{11} + 4W_{12} + 4W_{22}$. Then $\mathrm{Var}(Z)$ equals
(A)	150
(B)	200
(C)	250
(D)	300

Q.46	Consider the simple linear regression model $$y_i = \alpha + \beta x_i + \epsilon_i, i = 1, 2, \ldots, 24,$$ where $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}$ are unknown parameters, the errors $\epsilon_i's$ are i.i.d. random variables having $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is unknown. Suppose the following summary statistics are obtained from a data set of 24 observations $(x_1, y_1), \ldots, (x_{24}, y_{24})$: $$S_{xx} = \sum_{i=1}^{24} (x_i - \bar{x})^2 = 22.82, \ S_{yy} = \sum_{i=1}^{24} (y_i - \bar{y})^2 = 43.62,$$ $$\text{and } S_{xy} = \sum_{i=1}^{24} (x_i - \bar{x})(y_i - \bar{y}) = 15.48,$$ where $\bar{x} = \frac{1}{24}\sum_{i=1}^{24} x_i$ and $\bar{y} = \frac{1}{24}\sum_{i=1}^{24} y_i$. Then, for testing $H_0: \beta = 0$ against $H_1: \beta \neq 0$, the value of the F- test statistic based on the least squares estimator of β, whose distribution is $F_{1,22}$, equals (*rounded off to two decimal places*)
(A)	2.54
(B)	2.98
(C)	3.17
(D)	6.98

Q.47	Let $\{x_n\}_{n\geq1}$ be a sequence defined as $x_n = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} - 2(\sqrt{n} - 1)$. Then which of the following options is/are correct?
(A)	The sequence $\{x_n\}_{n\geq1}$ is unbounded
(B)	The sequence $\{x_n\}_{n\geq1}$ is monotonically decreasing
(C)	The sequence $\{x_n\}_{n\geq1}$ is bounded but does not converge
(D)	The sequence $\{x_n\}_{n\geq1}$ converges
Q.48	Let $\mathcal{O} = \{P : P$ is a 3×3 real matrix satisfying $P^T P = I_3$ and $\det(P) = 1\}$, where I_3 denotes the identity matrix of order 3. Then which of the following options is/are correct?
(A)	There exists a $P \in \mathcal{O}$ with $\lambda = \frac{1}{2}$ as an eigen value
(B)	There exists a $P \in \mathcal{O}$ with $\lambda = 2$ as an eigen value
(C)	If λ is the only real eigen value of $P \in \mathcal{O}$, then $\lambda = 1$
(D)	There exists a $P \in \mathcal{O}$ with $\lambda = -1$ as an eigen value

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Q.49	Let X_1, X_2, and X_3 be independent standard normal random variables, and let $Y_1 = X_1 - X_2$, $Y_2 = X_1 + X_2 - 2X_3$ and $Y_3 = X_1 + X_2 + X_3$. Then which of the following options is/are correct?
(A)	Y_1, Y_2 and Y_3 are independently distributed
(B)	$Y_1^2 + Y_2^2 + Y_3^2 \sim \chi_3^2$
(C)	$\dfrac{2Y_3}{\sqrt{3Y_1^2 + Y_2^2}} \sim t_2$
(D)	$\dfrac{3Y_1^2 + 2Y_3^2}{2Y_2^2} \sim F_{1,1}$
Q.50	Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables and $X_n \xrightarrow{a.s.} 0$ as $n \to \infty$. Then which of the following options is/are necessarily correct?
(A)	$\mathrm{E}(X_n^3) \to 0$ as $n \to \infty$
(B)	$X_n^7 \xrightarrow{P} 0$ as $n \to \infty$
(C)	For any $\epsilon > 0$, $\sum_{n=1}^{\infty} \Pr(\lvert X_n \rvert \geq \epsilon) < \infty$
(D)	$X_n^2 + X_n + 5 \xrightarrow{a.s.} 5$ as $n \to \infty$

Q.51	Consider a Markov chain $\{X_n : n = 1, 2, \dots\}$ with state space $S = \{1, 2, 3\}$ and transition probability matrix $$P = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/3 & 0 & 2/3 \\ 2/5 & 3/5 & 0 \end{pmatrix}.$$ Define $\pi = \left(\frac{18}{67}, \frac{24}{67}, \frac{25}{67}\right)$. Which of the following options is/are correct?
(A)	π is a stationary distribution of P
(B)	π^T is an eigen vector of P^T
(C)	$\Pr(X_3 = 1 \mid X_1 = 1) = \dfrac{11}{30}$
(D)	At least one state is transient
Q.52	Let $X_1, \dots, X_n$ be a random sample from a uniform distribution over the interval $\left(-\frac{\theta}{2}, \frac{\theta}{2}\right)$, where $\theta > 0$ is an unknown parameter. Then which of the following options is/are correct?
(A)	$2\max\{X_1, \dots, X_n\}$ is the maximum likelihood estimator of θ
(B)	$(\min\{X_1, \dots, X_n\},\ \max\{X_1, \dots, X_n\})$ is a sufficient statistic
(C)	$(\min\{X_1, \dots, X_n\},\ \max\{X_1, \dots, X_n\})$ is a complete statistic
(D)	$\frac{2(n+1)}{n}\max\{\lvert X_1\rvert, \dots, \lvert X_n\rvert\}$ is a uniformly minimum variance unbiased estimator of θ

Q.53	Let $X = (X_1, X_2, X_3)^T$ be a 3-dimensional random vector having multivariate normal distribution with mean vector $(0,0,0)^T$ and covariance matrix $$\Sigma = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 4 \end{pmatrix}.$$ Let $\alpha^T = (2, 0, -1)$ and $\beta^T = (1, 1, 1)$. Then which of the following statements is/are correct?
(A)	$\mathrm{E}\left(\mathrm{trace}(XX^T\alpha\alpha^T)\right) = 20$
(B)	$\mathrm{Var}\left(\mathrm{trace}(X\alpha^T)\right) = 20$
(C)	$\mathrm{E}\left(\mathrm{trace}(XX^T)\right) = 17$
(D)	$\mathrm{Cov}(\alpha^T X, \beta^T X) = 3$

Q.54	For $Y \in \mathbb{R}^n, X \in \mathbb{R}^{n\times p}$, and $\beta \in \mathbb{R}^p$, consider a regression model $$Y = X\,\beta + \epsilon,$$ where ϵ has an n-dimensional multivariate normal distribution with zero mean vector and identity covariance matrix. Let I_p denote the identity matrix of order p. For $\lambda > 0$, let $$\hat{\beta}_n = \left(X^T X + \lambda I_p\right)^{-1} X^T Y,$$ be an estimator of β. Then which of the following options is/are correct?
(A)	$\hat{\beta}_n$ is an unbiased estimator of β
(B)	$\left(X^T X + \lambda I_p\right)$ is a positive definite matrix
(C)	$\hat{\beta}_n$ has a multivariate normal distribution
(D)	$\mathrm{Var}(\hat{\beta}_n) = \left(X^T X + \lambda I_p\right)^{-1}$
Q.55	Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = x^2y^2 + 8x - 4y$. The number of saddle points of f is ____________ (*answer in integer*).

Q.56	Let $$P = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 & 1 \\ -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & -1 & 0 \end{pmatrix}.$$ If $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5 are eigen values of P, then $\prod_{i=1}^{5} \lambda_i$ equals ____________ (*answer in integer*).
Q.57	Let $P = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$. Then the value of trace$(P^5 + Q^4)$ equals ____________ (*answer in integer*).
Q.58	The moment generating functions of three independent random variables X, Y and Z are respectively given as $$M_X(t) = \frac{1}{9}(2 + e^t)^2, \quad t \in \mathbb{R},$$ $$M_Y(t) = e^{(e^t - 1)}, \quad t \in \mathbb{R},$$ $$\text{and} \quad M_Z(t) = e^{2(e^t - 1)}, \quad t \in \mathbb{R}.$$ Then $10 \Pr(X > Y + Z)$ equals __________ (*rounded off to two decimal places*).

Q.59	The service times (in minutes) at two petrol pumps P_1 and P_2 follow distributions with probability density functions $$f_1(x) = \lambda e^{-\lambda x},\ \ x > 0 \quad \text{and} \quad f_2(x) = \lambda^2 x\, e^{-\lambda x},\ \ x > 0,$$ respectively, where $\lambda > 0$. For service, a customer chooses P_1 or P_2 randomly with equal probability. Suppose, the probability that the service time for the customer is more than one minute, is $2e^{-2}$. Then the value of λ equals ____________ (*answer in integer*).
Q.60	Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables with $$\Pr\left(X_n = -\frac{1}{2^n}\right) = \Pr\left(X_n = \frac{1}{2^n}\right) = \frac{1}{2} \quad \forall\, n \in \mathbb{N}.$$ Suppose that $\sum_{i=1}^{n} X_i \xrightarrow{d} U$ as $n \to \infty$. Then $6\,\Pr\left(U \leq \frac{2}{3}\right)$ equals ____________ (*answer in integer*).
Q.61	Let $X_1, X_2, \ldots, X_7$ be a random sample from a population having the probability density function $$f(x) = \frac{1}{2}\lambda^3 x^2 e^{-\lambda x}\ ,\ \ x > 0\,,$$ where $\lambda > 0$ is an unknown parameter. Let $\hat{\lambda}$ be the maximum likelihood estimator of λ, and $\mathrm{E}(\hat{\lambda} - \lambda) = \alpha\lambda$ be the corresponding bias, where α is a real constant. Then the value of $\frac{1}{\alpha}$ equals ____________ (*answer in integer*).

Q.62	Let X_1, X_2 be a random sample from a population having probability density function $$f_\theta(x) = \begin{cases} e^{(x-\theta)} & \text{if } -\infty < x \le \theta \\ 0 & \text{otherwise,} \end{cases}$$ where $\theta \in \mathbb{R}$ is an unknown parameter. Consider testing $H_0: \theta \ge 0$ against $H_1: \theta < 0$ at level $\alpha = 0.09$. Let $\beta(\theta)$ denote the power function of a uniformly most powerful test. Then $\beta(\log_e 0.36)$ equals ____________ (*rounded off to two decimal places*).
Q.63	Let $X \sim Bin(3, \theta)$, where $\theta \in (0,1)$ is an unknown parameter. For testing $H_0: \frac{1}{4} \le \theta \le \frac{3}{4}$ against $H_1: \theta < \frac{1}{4}$ or $\theta > \frac{3}{4}$, consider the test $$\phi(x) = \begin{cases} 1 & \text{if } x \in \{0,3\} \\ 0 & \text{if } x \in \{1,2\}. \end{cases}$$ The size of the test ϕ is __________ (*rounded off to two decimal places*).
Q.64	Let $(X_1, X_2, X_3)^T$ have the following distribution $$N_3\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.4 & 0 \\ 0.4 & 1 & 0.6 \\ 0 & 0.6 & 1 \end{pmatrix}\right).$$ Then the value of the partial correlation coefficient between X_1 and X_2 given X_3 is __________ (*rounded off to two decimal places*).

Q.65	Let $(X,Y)^T$ follow a bivariate normal distribution with $\mathrm{E}(X)=2, \mathrm{E}(Y)=3$, $\mathrm{Var}(X)=16, \mathrm{Var}(Y)=25$, and $\mathrm{Cov}(X,Y)=14$. Then $$2\pi\left(\Pr(X>2, Y>3)-\frac{1}{4}\right)$$ equals ____________ (*rounded off to two decimal places*).