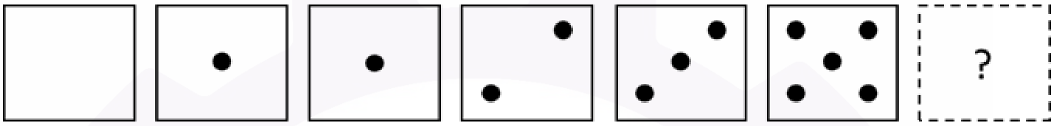
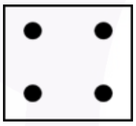
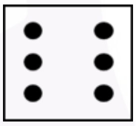
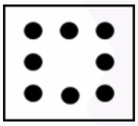
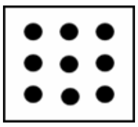
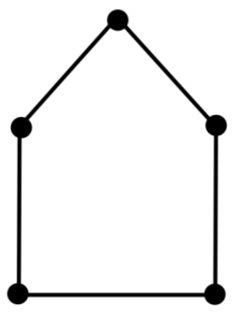
General Aptitude (GA)

Q.1 – Q.5 Carry ONE mark Each

Q.1	Suresh said, "I did it yesterday." Which one of the following options is the correct form of this sentence in indirect speech?
(A)	Suresh said that I did it yesterday.
(B)	Suresh says I did it yesterday.
(C)	Suresh says that he did it the day before.
(D)	Suresh said that he had done it the day before.

Q.2	To continue the sequence of tiles shown, the tile indicated by the question mark should be 
(A)	
(B)	
(C)	
(D)	

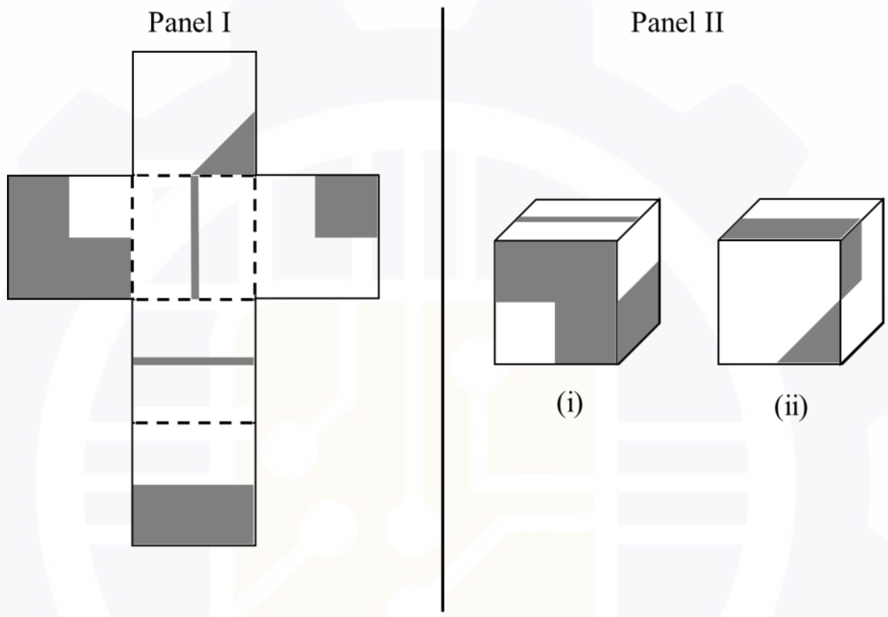
Q.3	Consider an art gallery whose walkways are shown as lines in the diagram. A black dot represents a junction of two walkways. A guard may be placed at a junction to watch over the walkways that join at that junction. The minimum number of guards needed to watch all the walkways is ________. 
(A)	2
(B)	3
(C)	4
(D)	5

Q.4	The 2nd of June is a Thursday in a certain year. Which day of the week is the 3rd of July in that year?
(A)	Thursday
(B)	Friday
(C)	Saturday
(D)	Sunday

Q.5	A coin with heads facing up is shown as (H) and a coin with tails facing up is shown as (T) . Six coins are placed in the Starting Arrangement, as shown in the figure below. A “step” is defined as interchanging a pair of adjacent coins without flipping them. The minimum number of steps needed to go from the Starting Arrangement to the Final Arrangement, as shown in the figure, is ________. 
(A)	3
(B)	6
(C)	9
(D)	12

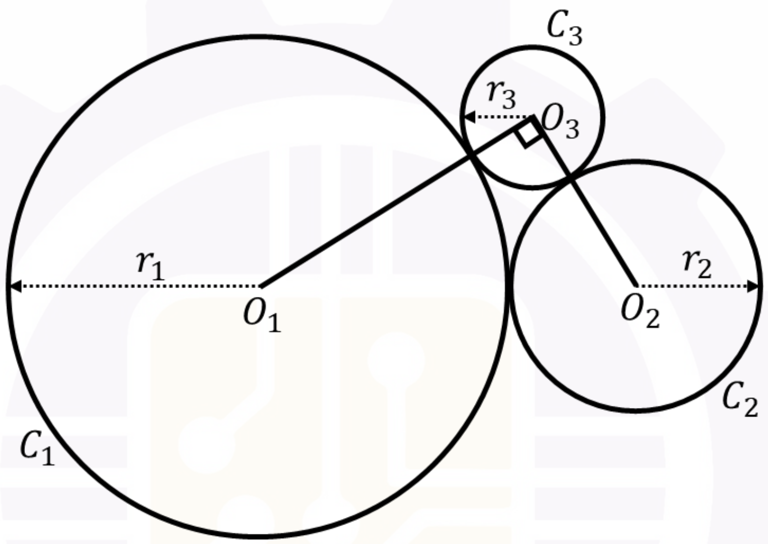
Q.6 – Q.10 Carry TWO marks Each

Q.6	Exacerbate : Mitigate :: ____________ Choose the option with the correct pair of words to fill the blank.
(A)	Aggravate : Alleviate
(B)	Alleviate : Precipitate
(C)	Aggravate : Precipitate
(D)	Emancipate : Exonerate

Q.7	A paper shown in Panel I is folded along the dashed lines (- - -) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to cubes shown in Panel II, which one of the options is correct? 
(A)	Only (i) can correspond to the unfolded cube in Panel I.
(B)	Only (ii) can correspond to the unfolded cube in Panel I.
(C)	Both (i) and (ii) can correspond to the unfolded cube in Panel I.
(D)	Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

Q.8	In a population, patients who have high cholesterol also have high blood-pressure (BP). Some patients with high BP also have diabetes. There are no patients who have both high cholesterol and diabetes. Furthermore, 1. the total number of patients with at least one of these conditions is 75, 2. the number of patients with high cholesterol is 10, 3. the number of patients with high BP is 45, and 4. the number of patients with only high BP and no other conditions is 20. Then the number of patients who have both diabetes and high BP is ________
(A)	0
(B)	15
(C)	20
(D)	10

Q.9	Four people P, Q, R, and S, of different ages, make the following observations. P – I am younger than S. Q – I am neither the youngest nor the oldest. R – P is older than me. Based on these observations, the youngest person is ______.
(A)	P
(B)	Q
(C)	R
(D)	S

Q.10	Circles C_1, C_2, and C_3, with centers O_1, O_2, and O_3 , and radii r_1 , r_2, and r_3, respectively, touch each other as shown in the following figure. Given $r_1 = 2$ cm, $r_2 = 1$ cm and the angle $\angle O_1O_3O_2$ is 90°, $r_3 =$ ______ cm. 
(A)	$\frac{1}{2}\left(-3+\sqrt{17}\right)$
(B)	$\frac{1}{2}\left(3+\sqrt{17}\right)$
(C)	$\frac{1}{2}\left(-2+\sqrt{17}\right)$
(D)	$\frac{1}{2}\left(-3+2\sqrt{17}\right)$

Q.11 – Q.35 Carry ONE mark Each

Q.11	Let $M_3(\mathbb{R})$ be the vector space of all 3×3 real matrices over $\mathbb{R}$ under usual matrix addition and scalar multiplication. Let $T_3(\mathbb{R})$ be the set of all 3×3 real upper triangular matrices. Which one of the following is TRUE?
(A)	The quotient space $M_3(\mathbb{R})/T_3(\mathbb{R})$ is isomorphic to the vector space of all 3×3 real skew symmetric matrices over $\mathbb{R}$ under usual matrix addition and scalar multiplication.
(B)	The quotient space $M_3(\mathbb{R})/T_3(\mathbb{R})$ is isomorphic to the vector space of all 3×3 real symmetric matrices over $\mathbb{R}$ under usual matrix addition and scalar multiplication.
(C)	The quotient space $M_3(\mathbb{R})/T_3(\mathbb{R})$ is isomorphic to the vector space of all 3×3 real lower triangular matrices over $\mathbb{R}$ under usual matrix addition and scalar multiplication.
(D)	The quotient space $M_3(\mathbb{R})/T_3(\mathbb{R})$ is isomorphic to the vector space of all 3×3 real matrices with trace zero over $\mathbb{R}$ under usual matrix addition and scalar multiplication.

Q.12	Let I be the integral defined as follows: $I = \int_0^1 \int_0^{\sqrt{y}} dx\, dy + \int_1^2 \int_{\sqrt{y-1}}^1 dx\, dy$ If the order of the integration is changed, then which one of the following is the correct expression for I?
(A)	$\int_0^{1/2} \int_{-x}^{x} dy\, dx + \int_{1/2}^{1} \int_{-x^2}^{-x^2+1} dy\, dx$
(B)	$\int_0^1 \int_{x^2}^{x^2+1} dy\, dx$
(C)	$\int_0^{1/2} \int_{x^2}^{x^2+1} dy\, dx + \int_{1/2}^{1} \int_{\sqrt{x}}^{\sqrt{x+1}} dy\, dx$
(D)	$\int_0^1 \int_{x^2}^{x^2-1} dy\, dx$

Q.13	Let C denote the Cantor set and $f:[0,1] \to \mathbb{R}$ be defined as follows: $$f(x) = \begin{cases} x^{2026} & \text{for } x \in C \\ \cos(\pi x) & \text{for } x \in \left[0, \frac{1}{2}\right] \setminus C \\ \sin(\pi x) & \text{for } x \in \left[\frac{1}{2}, 1\right] \setminus C \end{cases}$$ The value of the Lebesgue integral of $f(x)$ over the interval $[0,1]$ is equal to
(A)	$\frac{2}{\pi}$
(B)	$\frac{1}{\pi}$
(C)	$\frac{3}{\pi}$
(D)	0

Q.14	Let G be a group of order 595. Which one of the following is TRUE?
(A)	G cannot have a proper normal subgroup.
(B)	G must have a proper normal subgroup.
(C)	G cannot have an element of order 17.
(D)	The number of Sylow 5-subgroups is 17.

Q.15	Let $u(x,t)$ be the solution of the initial value problem for the heat equation on the real line: $$\frac{\partial u}{\partial t} = k\,\frac{\partial^2 u}{\partial x^2}\,, \qquad -\infty < x < \infty, \qquad t > 0, \qquad k \in \mathbb{R}$$ with the initial condition $$u(x,0) = e^{-a\|x\|}\,, \qquad a > 0.$$ If $\hat{u}(w,t) = \int_{-\infty}^{\infty} u(x,t)e^{iwx}\,dx$ is the Fourier transform of $u(x,t)$ with respect to x, then $\hat{u}(w,t)$ is equal to
(A)	$\frac{2a}{a^2+w^2}\,e^{-kw^2t}$
(B)	$\frac{a}{a^2+w^2}\,e^{-kw^2t}$
(C)	$\frac{2a}{a^2+w^2}\,e^{-ka^2t}$
(D)	$\frac{1}{\sqrt{4\,\pi kt}}\;e^{\frac{-w^2}{4kt}}$

Q.16	Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be defined as $$f(x, y) = (e^x \cos y\ , e^x \sin y)\ .$$ Which one of the following is TRUE?
(A)	f is one-to-one.
(B)	The Jacobian of f is negative.
(C)	f is locally invertible.
(D)	f is invertible on $\mathbb{R}^2$.

Q.17	Let $f: \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = \lvert z\rvert^2 - 5\bar{z} + 2$. Which one of the following is TRUE?
(A)	f is differentiable at $z = 5i$.
(B)	f is differentiable at $z = 5$.
(C)	f is differentiable at $z = -5i$.
(D)	f is differentiable at $z = -5$.

Q.18	Let $D = \{z \in \mathbb{C} \mid \|z\| < 1\}$ denote the unit disc in the complex plane $\mathbb{C}$. Let $f: D \to D$ be an analytic function which satisfies $f(0) = 0$. Then which one of the following is a possible value of $f'(0)$?
(A)	$\frac{5}{2}i$
(B)	$\frac{i}{10}$
(C)	$\frac{3}{2}$
(D)	$-\frac{5}{2}i$

Q.19	Let X be the space of all continuously differentiable real valued functions on $[0,1]$. Define the following norms on X: $$p_1(x) = \sup\{\lvert x(t)\rvert : t \in [0,1]\}$$ $$p_2(x) = \sup\left\{\left\lvert \frac{d}{dt}\, x(t)\right\rvert : t \in [0,1]\right\}$$ $$\text{and } \; p_3(x) = p_1(x) + p_2(x).$$ Which one of the following is TRUE?
(A)	(X, p_1) is a Banach space.
(B)	(X, p_2) is a Banach space.
(C)	(X, p_3) is not a Banach space.
(D)	(X, p_3) is a Banach space.

Q.20	Let X be a set with at least two elements. Let d_1, d_2 and d_3 be metrics on X. Which one of the following is NOT a metric on X?
(A)	$d(x,y) \coloneqq \min(d_1(x,y), 3)$ for all $x, y \in X$.
(B)	$d(x,y) \coloneqq \max(d_2(x,y), 3)$ for all $x, y \in X$.
(C)	$d(x,y) \coloneqq \frac{10\, d_3(x,y)}{1+d_3(x,y)}$ for all $x, y \in X$.
(D)	$d(x,y) \coloneqq \frac{1}{3}(d_1(x,y) + d_2(x,y) + d_3(x,y))$ for all $x, y \in X$.

Q.21	Let P be a 3×3 real symmetric positive definite matrix. Which of the following statements is/are TRUE?
(A)	$Q^\top PQ$ is positive definite for all nonzero $Q \in \mathbb{R}^{3\times 3}$.
(B)	$Q^\top PQ$ is positive semidefinite for all $Q \in \mathbb{R}^{3\times 3}$.
(C)	$Q^\top PQ$ is positive definite if $Q \in \mathbb{R}^{3\times 3}$ is nonsingular.
(D)	$Q^\top PQ$ is not positive semidefinite for some $Q \in \mathbb{R}^{3\times 3}$.

Q.22	Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a function defined as follows: $$f(x,y) = \begin{cases} \frac{x\sqrt{x^2+y^2}}{\lvert x \rvert} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$ Which of the following statements is/are TRUE?
(A)	$f(x,y)$ is continuous at $(0,0)$.
(B)	For any $u \in \mathbb{R}^2$, the directional derivative of f at $(0,0)$ in the direction of u exists.
(C)	$\frac{\partial f}{\partial x}(0,0) = 1 = \frac{\partial f}{\partial y}(0,0)$.
(D)	$f(x,y)$ is differentiable at $(0,0)$.

Q.23	Let $\mathbb{Z}_{11}$ be the ring of integers modulo 11. Let $\mathbb{Z}_{11}[x]$ be the ring of all polynomials with coefficients in $\mathbb{Z}_{11}$. Which of the following statements is/are TRUE?
(A)	The polynomial $x^2 + x + 4$ is irreducible over $\mathbb{Z}_{11}$.
(B)	$\mathbb{Z}_{11}[x] / \langle x^2 + x + 4 \rangle$ is a field.
(C)	The ideal $\langle x^2 + x + 4 \rangle$ is maximal.
(D)	The ideal $\langle x^2 + x + 4 \rangle$ is not a prime ideal.

Q.24	Consider $\mathbb{R}$ with the topology $$\tau = \{A \subseteq \mathbb{R} : A^c \text{ is finite}\} \cup \{\phi\}.$$ Which of the following statements is/are TRUE?
(A)	$(\mathbb{R}, \tau)$ is a Hausdorff space.
(B)	Any finite subset of $(\mathbb{R}, \tau)$ is closed.
(C)	$(\mathbb{R}, \tau)$ is compact.
(D)	$(\mathbb{R}, \tau)$ is connected.

Q.25	Consider the following partial differential equation: $$a\,\frac{\partial^2 f(x,y)}{\partial x^2} + b\,\frac{\partial^2 f(x,y)}{\partial y^2} = 8f(x,y)$$ where a and b are distinct positive real numbers. The combination(s) of the values of the real parameters ξ and η for which $f(x,y) = e^{2\xi x+\eta y}$ is a solution of the given partial differential equation, is/are
(A)	$\xi = \frac{1}{\sqrt{a}},\ \eta = \frac{2}{\sqrt{b}}$
(B)	$\xi = 0\ ,\ \eta = 0$
(C)	$\xi = \frac{1}{\sqrt{2a}},\ \eta = \frac{1}{\sqrt{2b}}$
(D)	$\xi = 0,\ \ \eta = 2\sqrt{\frac{2}{b}}$

Q.26	Consider the following assignment problem where X, Y, Z are tasks, P, Q, R are agents and the cost matrix is given by $$\begin{array}{c|ccc} & X & Y & Z \\ \hline P & 4 & 2 & 8 \\ Q & 2 & 3 & 7 \\ R & 3 & 1 & 6 \end{array}$$ Which of the following statements is/are TRUE for an optimal assignment?
(A)	The cost is 9.
(B)	Agent P is assigned to task Y.
(C)	Agent Q is assigned to task X.
(D)	Agent R is assigned to task Y.

Q.27	Consider the following differential equation: $$2x(x-2)^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+(x-2)y=0$$ Which of the following statements is/are TRUE ?
(A)	The point $x=2$ is not a singular point.
(B)	The point $x=2$ is a singular point.
(C)	The point $x=2$ is a regular singular point.
(D)	The point $x=2$ is not a regular singular point.
Q.28	Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be the linear transformation which reflects every vector in $\mathbb{R}^3$ through a two-dimensional subspace of $\mathbb{R}^3$. Let $P\in\mathbb{R}^{3\times 3}$ be the matrix representation of T using the basis $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$. Then the value of $2\times\text{trace}(P)-3\times\text{determinant}(P)$ is equal to ________ . (answer in integer)

Q.29	Let $x^3 - x + 1 \in \mathbb{Z}_3[x]$, where $\mathbb{Z}_3[x]$ is the ring of all polynomials with coefficients in $\mathbb{Z}_3$. Then the degree of the field extension $\mathbb{Z}_3[x] / \langle x^3 - x + 1 \rangle$ of $\mathbb{Z}_3$ is equal to ________. (answer in integer)
Q.30	Consider the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 1$. Using the modified Euler's method, the second approximation to $y(h)$, where $h = 0.05$ (step size), is equal to ________ . (rounded off to TWO decimal places)
Q.31	Let $$\alpha = \lim_{n\to\infty} \int_0^1 \frac{n^2 + (\sin e^x)^n}{7n^2 + x^8}\, dx\,.$$ The value of 14α is equal to ________ . (answer in integer)
Q.32	Let $X = (\mathbb{R}^3, \|\cdot\|_1)$ where $\left\| \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right\|_1 = \lvert x \rvert + \lvert y \rvert + \lvert z \rvert$ and $T: X \to X$ be a linear transformation defined as $$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & -2 \\ 1 & 3 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$ Then the operator norm of T is equal to ________. (answer in integer)

Q.33	Consider the following Linear Programming Problem (LPP): $$\text{maximize } z = 10x_1 + 20x_2$$ subject to $$x_1 \leq 36$$ $$x_2 \geq 42$$ $$x_1 + x_2 \geq 48$$ $$5x_1 + x_2 \leq 150$$ $$x_1, x_2 \geq 0$$ The maximum value of z in the above LPP is equal to ________. (answer in integer)
Q.34	The number of distinct topologies on the set $\{1, 2, 3\}$ consisting of exactly four elements is equal to ________ . (answer in integer)
Q.35	For $a_n \in \mathbb{C}$, $n = 0,1,2, \ldots$, the power series $$\sum_{n=0}^{\infty} a_n (z-2)^n$$ converges at $z = 5$ and diverges at $z = -1$. Then the radius of convergence of this power series is equal to ________. (answer in integer)

Q.36 – Q.65 Carry TWO marks Each

Q. 36	The solution $y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$ of the initial value problem $$\frac{dy}{dx} = \begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix} y, \qquad y(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ is equal to
(A)	$3\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{x} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-x}$
(B)	$-3\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{x} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-x}$
(C)	$3\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{x} + 2\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-x}$
(D)	$3\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{x} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-x}$

Q.37	Let $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{pmatrix}$. Define an inner product on $\mathbb{R}^3$ with respect to P as $$\langle x, y \rangle_P = x^\top P y, \quad \text{for all } x, y \in \mathbb{R}^3.$$ Consider the subspace $V = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$. Which one of the following sets is an orthonormal basis of V with respect to $\langle x, y \rangle_P$?
(A)	$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$
(B)	$\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$
(C)	$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{1}{5} \\ \frac{1}{5} \end{pmatrix} \right\}$
(D)	$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \right\}$

Q. 38	The value of the contour integral $$\int_{\Gamma} \frac{(1-z)(3\cos z + 5\sin z)}{1 - z^{101}}\, dz\,,$$ where $\Gamma = \left\{ z \in \mathbb{C} : \lvert z \rvert = \frac{7}{9} \right\}$, oriented in the counter-clockwise direction, is equal to
(A)	$2\pi i$
(B)	$-2\pi i$
(C)	0
(D)	$4\pi i$

Q. 39	Let $f(z) = \sum_{n=1}^{\infty} 5^{-n} \cos(nz)$. On which one of the following domains, does $f(z)$ represent an analytic function?
(A)	$\{z \in \mathbb{C} : \lvert \mathrm{Im}\, z \rvert < \ln 5\}$
(B)	$\{z \in \mathbb{C} : \lvert \mathrm{Im}\, z \rvert > \ln 5\}$
(C)	$\{z \in \mathbb{C} : \lvert \mathrm{Re}\, z \rvert < \ln 5\}$
(D)	$\{z \in \mathbb{C} : \lvert \mathrm{Re}\, z \rvert > \ln 5\}$

Q. 40	Let $\delta(t)$ be the unit impulse function defined by $\delta(x-x_0) = 0$ when $x \neq x_0$ and $$\int_{-\infty}^{\infty} \delta(x-x_0)\,dx = 1.$$ Consider the following initial value problem $$2\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = \delta(x-5)$$ with $y(0) = 0$ and $\frac{dy}{dx}(0) = 0$. Which one of the following is TRUE?
(A)	$y(10) = \frac{10}{\sqrt{15}} e^{-5/4} \sin\left(\frac{\sqrt{15}}{4}\right)$
(B)	$y(10) = \frac{2}{\sqrt{15}} e^{-5/4} \sin\left(\frac{5\sqrt{15}}{4}\right)$
(C)	$y(10) = \frac{10}{\sqrt{15}} e^{5/4} \sin\left(\frac{\sqrt{15}}{4}\right)$
(D)	$y(10) = \frac{2}{\sqrt{15}} e^{5/4} \sin\left(\frac{5\sqrt{15}}{4}\right)$

Q.41	Which one of the following statements is TRUE?
(A)	The set of all 2×2 real matrices under usual matrix addition and multiplication is a Unique Factorization Domain.
(B)	The set of all real valued functions under usual function addition and multiplication is a Unique Factorization Domain.
(C)	The set of all polynomials with coefficients in $\mathbb{R}$ under usual polynomial addition and multiplication is a Unique Factorization Domain.
(D)	The set of Riemann integrable functions in $[0,1]$ under usual function addition and multiplication is a Unique Factorization Domain.

Q. 42	Let $\ell^\infty = \{x = (x_n)_{n\geq 1} \mid x_n \in \mathbb{R}, \sup\{\lvert x_n \rvert : n = 1,2,\ldots\} < \infty\}$ with the supremum norm. Let $T: \ell^\infty \to \ell^\infty$ be given by $T(x_1, x_2, x_3, \ldots) = \left(x_1, \frac{x_2}{2}, \frac{x_3}{3}, \ldots\right)$. Which one of the following is TRUE?
(A)	T is bounded but not one-to-one.
(B)	T is one-to-one but not bounded.
(C)	T is bounded and the inverse (from the range of T) exists but not bounded.
(D)	T is bounded and the inverse (from the range of T) is bounded.

Q. 43	Let $u(x,t)$ satisfy the wave equation $$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0, \quad c > 0$$ with $$u(x,0) = \begin{cases} 1 & \text{if } \lvert x\rvert < 1 \\ 0 & \text{otherwise} \end{cases}$$ and $$\frac{\partial u}{\partial t}(x,0) = 0.$$ By using D'Alembert's formula, the maximum value of $u(0,t)$ for $t > 0$ is
(A)	1
(B)	$\frac{1}{2}$
(C)	$\frac{1}{4}$
(D)	0

Q. 44	Consider the Laplace equation $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0,\ 0 < x < 1,\ 0 < y < 1$, with the boundary conditions $T(x,0) = x, \quad T(0,y) = y$ $T(x,1) = 1 + x, \quad T(1,y) = 1 + y.$ Then the value of $T\left(\frac{1}{2},\frac{1}{3}\right)$ is equal to
(A)	$\frac{7}{6}$
(B)	$\frac{5}{6}$
(C)	$\frac{1}{6}$
(D)	$\frac{1}{2}$

Q.45	Let E_1 and E_2 be subsets of a normed linear space X, and $$E_1 + E_2 = \{x + y : x \in E_1, y \in E_2\}$$ $$E_1 \times E_2 = \{(x, y) : x \in E_1, y \in E_2\}.$$ Which one of the following is NOT TRUE?
(A)	If either of E_1 or E_2 is open, then $E_1 + E_2$ is open.
(B)	If E_1 and E_2 are convex, then $E_1 + E_2$ is convex.
(C)	If $E_1$and E_2 are closed, then $E_1 + E_2$ is closed.
(D)	If $E_1$and E_2 are connected, then $E_1 \times E_2$ is connected.

Q. 46	For a transportation problem, let c_{ij} denote the unit cost of the cell (i, j). Assume that $c_{11} = 10,\ c_{12} = 12,\ c_{21} = 11$ and $c_{22} = 13$. Let α_i and β_j, $i, j = 1,2,3$, represent the simplex multipliers associated with any basis corresponding to the unit cost c_{ij}. Assume that $\alpha_1 = x,\ \alpha_2 = x + 1,\ \beta_1 = y$ and $\beta_2 = y + 2$. The relative cost coefficient d_{ij} is the difference between the current solution and the new improved solution. If $x = 4,\ c_{13} = 19$ and $\beta_3 = y + 5$, then which one of the following is TRUE?
(A)	$y = 6$ and $d_{13} = 7$
(B)	$y = 6$ and $d_{13} = 4$
(C)	$y = 5$ and $d_{13} = 7$
(D)	$y = 5$ and $d_{13} = 4$
Q. 47	Let X and Y be topological spaces and $f: X \rightarrow Y$ be a continuous and bijective mapping. Which one of the following statements is TRUE?
(A)	f is a homeomorphism if X and Y are compact.
(B)	f is a homeomorphism if X is Hausdorff and Y is compact.
(C)	f is a homeomorphism if X is compact and Y is Hausdorff.
(D)	f is a homeomorphism if X and Y are Hausdorff.

Q.48	Let P and Q be 3×3 nonzero real matrices. Assume that there exists a 3×3 real nonsingular matrix S such that $S^{-1}PS$ and $S^{-1}QS$ are both upper triangular. Which of the following statements is/are TRUE?
(A)	The matrix $PQ - QP$ is invertible.
(B)	The matrix $PQ - QP$ is nilpotent.
(C)	The matrix $I + PQ - QP$ is invertible, where I is the 3×3 identity matrix.
(D)	If $PQ - QP$ is a nonzero matrix, then $PQ - QP$ is diagonalizable.
Q. 49	Let $S = \bigcup_{n=1}^{\infty} \left\{(x, y) \in \mathbb{R}^2 : (x - n)^2 + y^2 = \frac{1}{n^2}\right\}$ with usual topology. Which of the following statements is/are TRUE?
(A)	S is not compact.
(B)	S is compact.
(C)	S is connected.
(D)	S is not connected.

Q. 50	Let $f(z) = \frac{z}{1-z}$ and $g(z) = \frac{1+z}{1-z}$ be two Möbius transformations defined on the unit disc $D = \{z \in \mathbb{C} : \lvert z \rvert < 1\}$. Consider the following statements: S_1: $f(D) \subseteq g(D)$ S_2: $g(D) \subseteq f(D)$ Which of the following statements is/are CORRECT?
(A)	S_1 is true.
(B)	S_2 is true.
(C)	S_2 is true and S_1 is false.
(D)	Neither S_1 is true nor S_2 is true.

Q. 51	Let α and β be the roots of the indicial equation obtained in the method of finding Frobenius series solution to the differential equation: $$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = 0\,.$$ Which of the following statements is/are TRUE?
(A)	$\alpha \neq \beta$
(B)	$\alpha - \beta \in \mathbb{Z}$
(C)	$\alpha - \beta \notin \mathbb{Z}$
(D)	$\alpha = \beta$

Q.52	Let $\mathbb{Q}[x]$ be the ring of all polynomials with coefficients in $\mathbb{Q}$ under the usual polynomial addition and multiplication. Let $T: \mathbb{Q}[x] \to \mathbb{Q}[x]$ be defined by $$T(p(x)) = p(x^2), \text{ for all } p(x) \in \mathbb{Q}[x].$$ Which of the following statements is/are TRUE?
(A)	T is a ring homomorphism.
(B)	T is one-to-one.
(C)	T is onto.
(D)	T is a ring isomorphism.

Q. 53	Let X be any normed linear space and X' be the dual space of X. Which of the following statements is/are TRUE?
(A)	If X is separable and X' is non-separable, then X must be reflexive.
(B)	If X is separable and X' is non-separable, then X cannot be reflexive.
(C)	If X is reflexive, then X' is reflexive.
(D)	If X' is separable, then X is separable.
Q. 54	Let $(\mathbb{Z}_n, +)$ be the group of integers modulo n. Let $G = \mathbb{Z}_{30} \oplus \mathbb{Z}_{12}$ be the external direct product of $\mathbb{Z}_{30}$ and $\mathbb{Z}_{12}$. Then which of the following statements is/are TRUE?
(A)	G is not a cyclic group.
(B)	The order of the element $(14,7)$ in G is 98.
(C)	The order of the element $(14,7)$ in G is 60.
(D)	There is an element in G of order 360.

Q.55	Let $J = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$. Then the geometric multiplicity of the eigenvalue 2 of J is equal to ________ . (answer in integer)
Q.56	Let $\alpha = \iint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = (2x + 3z)\hat{\imath} + (xz - y)\hat{\jmath} + (y^2 + 2z)\hat{k}$ and S is the sphere with centre at $(3, -1, 2)$ and radius 9. Here, $\hat{n}$ is the unit normal drawn outward and $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors. Then the value of $\frac{1}{36\,\pi}\alpha$ is equal to ________. (answer in integer)
Q.57	Consider the problem of maximizing $z = (x_1 \quad x_2 \quad x_3)\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ subject to $(x_1 \quad x_2 \quad x_3)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 1, \qquad \text{where } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3.$ Then the maximum value of z is ________. (answer in integer)

Q.58	Let $P_3(\mathbb{R})$ be the vector space of all polynomials of degree at most three with the real coefficients under usual polynomial addition and scalar multiplication. Let $T: P_3(\mathbb{R}) \to \mathbb{R}^2$ be the linear transformation defined as $$T(p) = \big(p(1), p'(1)\big)$$ for all $p \in P_3(\mathbb{R})$, where p' is the derivative of p. Then the nullity of T is equal to ________. (answer in integer)
Q. 59	The number of zeros of the complex polynomial $z^6 + 5z^3 + 4z + 11$ in the annulus $\{z \in \mathbb{C} : 1 < \lvert z \rvert < 3\}$ is equal to ________ . (answer in integer)
Q. 60	Let $L^2[0, \pi]$ denote the space of all real valued Lebesgue square integrable functions on $[0, \pi]$. Let $T: L^2[0, \pi] \to L^2[0, \pi]$ be defined as follows: $$T\big(f(x)\big) = \sin x \int_0^{\pi} f(t) \cos t \, dt + \cos x \int_0^{\pi} f(t) \sin t \, dt \, .$$ Then the value of $\frac{4}{\pi} \lVert T \rVert$ is equal to ________ . (answer in integer)
Q. 61	Let $\Omega = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ be the open unit disc and $\partial\Omega$ be its boundary. If $u(x, y)$ is the solution of the following Dirichlet problem $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{in} \quad \Omega$$ $$u(x, y) = x^2 - y^2 \quad \text{on} \quad \partial\Omega,$$ then the value of $4\left(u\left(\frac{1}{2}, 0\right) - u\left(0, \frac{1}{2}\right)\right)$ is ________ . (answer in integer)
Q. 62	Let $X = \{5, 6, 7, 8, 9, 10\}$ be equipped with the topology $$\tau = \{\phi, X, \{5,6,7\}, \{8,9,10\}\}.$$ Then the number of subsets of X which are neither open nor closed is ________ . (answer in integer)

Q. 63	Let $\alpha, \beta \in \mathbb{R}$. If $(4, 0, 2, \beta)$ is an optimal solution of the Linear Programming Problem: $$\text{minimize} \quad x_1 + 3x_2 + 2x_3 - \alpha x_4$$ subject to $$4x_1 + x_2 + x_3 = 18$$ $$-3x_1 + 2x_3 + x_4 = 2$$ $$x_1, x_2, x_3, x_4 \geq 0,$$ then the maximum value of $22(\alpha + \beta)$ is equal to ________ . (answer in integer)
Q. 64	Let $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2, 0 \leq y \leq 2\}$ and let $f(t)$ denote the smallest integer greater than or equal to t. Then the value of the integral $$\iint_D f(x + y)\,dxdy$$ is ________ . (answer in integer)
Q. 65	If Jacobi method is used to solve the following system of linear equations $$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$ with the initial guess $x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $x^{(i)} = \begin{pmatrix} x_1^{(i)} \\ x_2^{(i)} \\ x_3^{(i)} \end{pmatrix}, i = 1,2,3 \ldots$, denotes the i^{th} iterate, then the value of $\left| x_1^{(2)} + x_2^{(2)} + x_3^{(2)} \right|$ is equal to ________. (answer in integer)