## General Aptitude (GA)

### Q.1 – Q.5 Carry ONE mark Each

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| Q.1 | The antonym of the word protagonist is ________. |
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| (A) | agnostic |
| (B) | antagonist |
| (C) | arsonist |
| (D) | anarchist |
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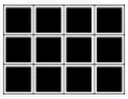
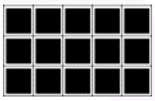
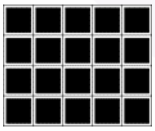
| Q.2 | The figure shows two 4-tile patterns.<br><br>Either one or both of the patterns can be used any number of times and in any orientation to construct a new pattern. Which one of the options below **cannot** be constructed by using only these two 4-tile patterns assuming there are no overlaps among them? |
| | |
| (A) |  |
| (B) |  |
| (C) |  |
| (D) |  |
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| Q.3 | Consider a knock-out women’s badminton singles tournament where there are no ties. The loser in each game is eliminated from the tournament. Every player plays until she is defeated or remains the last undefeated player. The last undefeated player is declared the winner of the tournament. If there are 64 players in the beginning of the tournament, how many games should be played in total to declare the winner of the tournament? |
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| (A) | 127 |
| (B) | 64 |
| (C) | 63 |
| (D) | 32 |
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| Q.4 | A student needs to enroll for a minimum of 60 credits. A student cannot enroll for more than 70 credits. The credits are divided amongst project and three distinct sets of courses namely, core courses, specialization courses, and elective courses. It is compulsory for a student to enroll for exactly 15 credits of core courses and exactly 20 credits of project. In addition, a student has to enroll for a minimum of 10 credits of specialization courses. The maximum credits of elective courses that a student can enroll for is ______ |
| | |
| (A) | 10 |
| (B) | 15 |
| (C) | 20 |
| (D) | 25 |
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| Q.5 | ‘When the teacher is in the room, all students stand silently.’<br><br>If the above statement is true, which one of the following statements is **not necessarily** true? |
| | |
| (A) | If any student is not standing silently, then the teacher is not in the room. |
| (B) | When the teacher is in the room, all students are silent. |
| (C) | If all students are standing, then the teacher is in the room. |
| (D) | When the teacher is in the room, all students are standing. |
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## Q.6 – Q.10 Carry TWO marks Each

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| Q.6 | Combinatorics deals with problems involving counting. For example, "How many distinct arrangements of N distinct objects in M spaces on a circle are possible?" is a typical problem in combinatorics. This kind of counting is sometimes used in the modeling of several physical phenomena. Often, in such models, the different combinatorial possibilities are assigned probability values. Assigning probabilities enables the computation of the average values of physical quantities.<br><br>Consider the following statements:<br><br>P: Combinatorics is always invoked in the modeling of physical phenomena.<br><br>Q: Modeling some physical phenomena involves assigning probabilities to combinatorial possibilities in order to compute average values of physical quantities.<br><br>Based on the passage above, what can be inferred about statements P and Q? |
| | |
| (A) | P is False and Q is False |
| (B) | P is False and Q is True |
| (C) | P is True and Q is False |
| (D) | P is True and Q is True |
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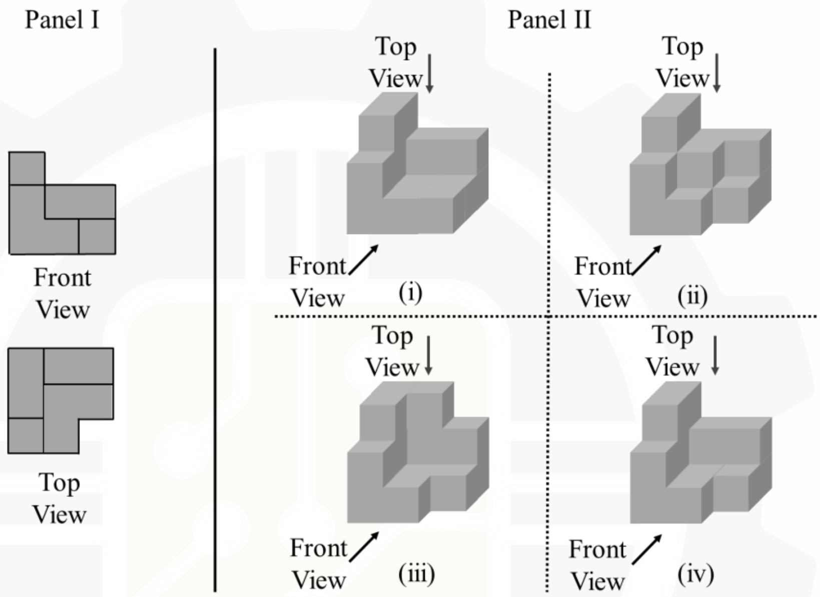
| Q.7 | In Panel I of the figure below, the front view and top view of a structure are shown. Which one of the 3D structures shown in Panel II possesses the views shown in Panel I?<br><br> |
| | |
| (A) | (i) |
| (B) | (ii) |
| (C) | (iii) |
| (D) | (iv) |
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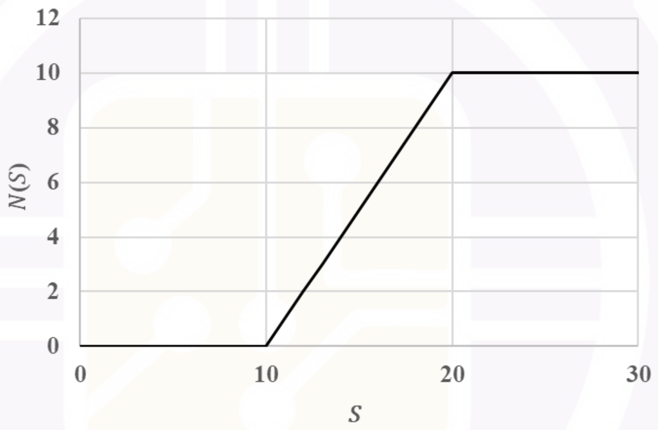
| Q.8 | For positive real numbers $S$ and $K$, the function $H_K(S)$ is defined as: $H_K(S) = \max(S-K, 0)$. The max function is defined as: $$\max(a,b) = \begin{cases} a, & \text{when } a > b \\ b, & \text{when } a \le b \end{cases}$$ The graph below shows the plot of a function $N(S)$ versus $S$. $N(S)$ can be expressed as ______.   |
| | |
| (A) | $H_{10}(S) - H_{20}(S)$ |
| (B) | $H_{10}(S) - 2H_{20}(S)$ |
| (C) | $-H_{10}(S) + H_{20}(S)$ |
| (D) | $H_{15}(S) - H_{20}(S)$ |
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| Q.9 | In the 2020 summer Olympics' Javelin throw finals, Neeraj Chopra exhibited a spectacular performance to win the gold medal. The silver medal was won by Jakub Vadlejch and the bronze medal was won by Vitezlav Vesely. There were six rounds of throws with each athlete having one throw per round. The best of all the throws of each athlete is considered for the medal. Following were the observations about the throws:<br><br>i. The first and second rounds were dominated by Neeraj Chopra with a gold medal performance in his second throw, while the other two athletes did not have any medal winning throws in these rounds.<br><br>ii. The throws in the last round by both Jakub Vadlejch and Vitezlav Vesely were fouls and were not considered for scoring.<br><br>iii. After four rounds, Vitezlav Vesely was in the second position and could not improve upon his best throw in the succeeding rounds.<br><br>iv. In the fourth round, the throw by Jakub Vadlejch was the best in that round.<br><br>In which round did Vitezlav Vesely have his best throw? |
| | |
| (A) | Third |
| (B) | Fourth |
| (C) | Fifth |
| (D) | Sixth |
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| Q.10 | An unbiased six-faced dice whose faces are marked with numbers 1, 2, 3, 4, 5, and 6 is rolled twice in succession and the number on the top face is recorded each time. The probability that the number appearing in the second roll is an integer multiple of the number appearing in the first roll is ____________ |
| | |
| (A) | $\frac{1}{6}$ |
| (B) | $\frac{5}{18}$ |
| (C) | $\frac{7}{18}$ |
| (D) | $\frac{5}{6}$ |
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## Q.11 – Q.35 Carry ONE mark Each

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| Q.11 | Consider the polynomial $$p(t) = t(t-1)(t-2)$$ and define $$F(x) = \int_{\frac{1}{2}}^{x} \frac{1}{p(t)}\,dt, \qquad x \in \left[\frac{1}{2}, 1\right).$$ Which of the following statements is correct? |
| | |
| (A) | $F$ is strictly decreasing |
| (B) | $F$ is strictly increasing |
| (C) | $\lim_{x \to 1} F(x)$ exists |
| (D) | $F'\left(\frac{3}{4}\right) = 1$ |
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| Q.12 | Let $f: (-1, 1) \to \mathbb{R}$ be a differentiable function. Consider the following statements:<br><br>(I) Suppose $f(0) \geq 0$, and $f'(x) > 0$ whenever $f(x) = 0$, for any $x \geq 0$. Then $f(x) > 0$, for any $x > 0$.<br>(II) Suppose $f(0) \leq 0$, and $f'(x) > 0$ whenever $f(x) = 0$, for any $x \leq 0$. Then $f(x) < 0$, for any $x < 0$.<br><br>Which of the following statements is correct? |
| | |
| (A) | Only statement (I) is correct |
| (B) | Only statement (II) is correct |
| (C) | Both statements (I) and (II) are correct |
| (D) | Neither statement (I) nor statement (II) is correct |
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| Q.13 | Consider the following two subspaces of $\mathbb{R}^4$ $$W_1 = \{(x_1, x_2, x_3, x_4) :\ x_1 + x_2 + x_3 + x_4 = 0\}$$ $$W_2 = \{(x_1, x_2, x_3, x_4) : x_1 + 2x_2 + 3x_3 + 4x_4 = 0\}.$$ Which of the following statements is correct? |
| | |
| (A) | $\dim(W_1 \cap W_2) = 0$ |
| (B) | $\dim(W_1 \cap W_2) = 1$ |
| (C) | $\dim(W_1 \cap W_2) = 2$ |
| (D) | $\dim(W_1 \cap W_2) = 3$ |
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| Q.14 | Let $X$ and $Y$ be two independent discrete random variables such that the moment generating functions of $X$ and $X+Y$ are given by $$M_X(t) = \frac{1+2e^{-t}+3e^{2t}}{6}, \quad t \in \mathbb{R},$$ and $$M_{X+Y}(t) = \frac{2+e^{t}+3e^{3t}}{6}, \quad t \in \mathbb{R},$$ respectively. Then which of the following statements is correct? |
| | |
| (A) | $P(XY=0)=\frac{1}{6}$ |
| (B) | $P(XY=2)=\frac{1}{6}$ |
| (C) | $E(X)=0$ |
| (D) | $\mathrm{Var}(Y)=1$ |
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| Q.15 | Let $X$ and $Y$ be identically distributed random variables with variance $\sigma^2 \in (0, \infty)$. Then the correlation coefficient between $X$ and $Y$ is |
| | |
| (A) | $1 - \frac{E(X-Y)^2}{2\,\sigma^2}$ |
| (B) | $1 - \frac{2\,E(X-Y)^2}{\sigma^2}$ |
| (C) | $1 - \frac{E(X-Y)^2}{\sigma^2}$ |
| (D) | $1 - \frac{E(X+Y)^2}{\sigma^2}$ |
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| Q.16 | Let $X$ and $Y$ be independent and identically distributed normal random variables. If $$P(X+2Y \leq 3) = P(2X - Y \geq 4),$$ then $E(X)$ is |
| | |
| (A) | $\frac{7}{3}$ |
| (B) | $\frac{7}{4}$ |
| (C) | $\frac{3}{7}$ |
| (D) | $\frac{4}{7}$ |
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| Q.17 | Let $X$ be a random variable with the following probability density function $$f(x) = \begin{cases} 4x^2\, e^{-2x} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$ If $Y = \ln X$, then which of the following statements is correct? |
| | |
| (A) | $E(Y)$ is not finite |
| (B) | $E(Y) = \ln\frac{3}{2}$ |
| (C) | $E(Y) > \ln\frac{3}{2}$ |
| (D) | $E(Y) < \ln\frac{3}{2}$ |
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| Q.18 | Let $\{X_n : n \geq 0\}$ be a homogeneous Markov chain with state space $S = \{1, 2, \ldots, 7\}$ and transition probability matrix $$P = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{2}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}.$$ Then which of the following statements is correct? |
| | |
| (A) | 2 is a transient state |
| (B) | 3 is a transient state |
| (C) | 4 is a transient state |
| (D) | 5 is a transient state |
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| Q.19 | Let $X_1, X_2$ be a random sample from the following probability density function $$f_\alpha(x) = \begin{cases} \alpha x^{\alpha-1} e^{-x^\alpha} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$ where $\alpha \in (0, \infty)$ is an unknown parameter. Let $$X_{(1)} = \min\{X_1, X_2\} \quad \text{and} \quad X_{(2)} = \max\{X_1, X_2\}.$$ Then which of the following statements is correct? |
| | |
| (A) | $(X_1 + X_2,\ X_{(1)})$ is a minimal sufficient statistic |
| (B) | $(X_1,\ X_{(2)})$ is a minimal sufficient statistic |
| (C) | $(X_1 - X_2,\ X_{(1)})$ is a sufficient statistic but not a minimal sufficient statistic |
| (D) | $(X_1 - X_2,\ X_{(2)})$ is a complete sufficient statistic |
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| Q.20 | Let $X_1$ and $X_2$ be independent and identically distributed random variables following normal distribution with mean $\theta \in (-\infty, \infty)$ and variance 1. Then which of the following estimators of their expected values attains the Cramer-Rao lower bound? |
| | |
| (A) | $X_1 + X_2$ |
| (B) | $\frac{X_1^2+X_2^2+5}{2}$ |
| (C) | $(X_1 + X_2)^2$ |
| (D) | $2X_1 - X_2$ |
| | |
| Q.21 | For testing a null hypothesis $H_0$ against an alternative hypothesis $H_1$ at level of significance $\alpha \in (0, 1)$, which of the following statements is correct? |
| | |
| (A) | A uniformly most powerful test always exists |
| (B) | If a uniformly most powerful test exists then its size is $\alpha$ |
| (C) | If a uniformly most powerful unbiased test exists then it is necessarily a uniformly most powerful test |
| (D) | If a uniformly most powerful test exists then it is necessarily an unbiased test |
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| Q.22 | Let $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n), n \geq 2$, be a random sample from a continuous bivariate distribution with joint distribution function $F_{X,Y}$. Further, $F_X$ and $F_Y$ are the marginal distribution functions of $X$ and $Y$, respectively. If $$F_{X,Y}(x, y) = F_X(x)F_Y(y), \ \forall\ (x, y),$$ then, for any two independent pairs $(X_i, Y_i)$ and $\left(X_j, Y_j\right)$, $$P\left[\left(X_i - X_j\right)\left(Y_i - Y_j\right) > 0\right]$$ equals |
| | |
| (A) | $\frac{1}{4}$ |
| (B) | $\frac{1}{2}$ |
| (C) | $\frac{3}{4}$ |
| (D) | $\frac{3}{8}$ |
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| Q.23 | For a random sample $X_1, \dots, X_n, n \geq 2$, from a population with distribution function $F_X$, let $Z_n(x)$ be the proportion of sample values less than or equal to $x$, $x \in \mathbb{R}$. Which of the following statements is/are true? |
| | |
| (A) | $E(Z_n(x)) = F_X(x)$ |
| (B) | $\text{Var}(Z_n(x)) = \frac{F_X(x)(1-F_X(x))}{n^2}$ |
| (C) | $\text{Cov}(Z_n(x), Z_n(y)) = 0$, for all $x \neq y$ |
| (D) | $Z_n(x)$ is a consistent estimator of $F_X(x)$ |
| | |
| Q.24 | The value of $$\lim_{n\to\infty} n \int_{1-\frac{1}{2n}}^{1+\frac{1}{2n}} e^{t^2-1}\,dt$$ equals ___________ *(answer in integer).* |
| Q.25 | Consider the system of linear equations $$x + y + z = 1$$ $$2x + y + 3z = 6$$ $$3x + 2y + kz = k + 1$$ If the above system has no solution, then the value of $k$ equals ______________ *(answer in integer).* |

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| Q.26 | Consider the subspace $$U = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_2 + x_4 = 0,\ x_3 = 0\}.$$ Let $U^{\perp}$ be its orthogonal complement. The value of $\dim(U^{\perp})$ equals ________ *(answer in integer).* |
| Q.27 | Let $X$ be a random variable such that $$P(X = i) = 2P(X = i - 1),\ \ i = 2, 3, \dots, n,\ \ n \geq 7,$$ and $\sum_{i=1}^{n} P(X = i) = 1$. Then $(2^n - 1)P(X = 7)$ equals ________ *(answer in integer).* |
| Q.28 | Let $X$ and $Y$ be two continuous random variables having the following joint probability density function $$f(x, y) = \begin{cases} x + y & \text{if } 0 < x < 1,\ \ 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$ Then $72\left(\mathrm{Var}(X) + \mathrm{Var}(Y)\right)$ equals ________ *(answer in integer).* |
| Q.29 | Let $X_1, X_2$ and $X_3$ be three independent random variables such that $X_k$ $(k = 1, 2, 3)$ has the following probability density function $$f_k(x) = \begin{cases} k\, e^{-kx} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$ Let $Y = \min\{X_1,\ X_2,\ X_3\}$. Then the value of $E(3Y^2 - Y)$ equals ________ *(answer in integer).* |
| Q.30 | Let $X$ be a continuous random variable having distribution function $F$. If $$Y = -3 \ln F(X),$$ then $E(Y)$ equals ________ *(answer in integer).* |

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| Q.31 | Let $\{W(t) : t \geq 0\}$ be a standard Brownian motion, with $W(0) = 0$. Define $$Z_1 = W(1) + W(2) \quad \text{and} \quad Z_2 = W(2) + W(3).$$ Let $\rho$ be the correlation coefficient between $Z_1$ and $Z_2$. Then the value of $10\rho$ is __________ *(round off to two decimal places).* |
| Q.32 | Let $x_1, x_2, \ldots, x_n$ $(n \geq 2)$ be the observed values of a random sample from the following probability density function $$f(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$ where $\alpha \in (0, \infty)$ and $\lambda \in (0, \infty)$ are unknown parameters. If $$\frac{x_1 + x_2 + \cdots + x_n}{n} = 2 \quad \text{and} \quad \frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n} = 5,$$ then the method of moments estimate of $\alpha$ equals ________ *(answer in integer).* |
| Q.33 | Let $X_1, X_2, \ldots, X_{10}$ be a random sample from the following probability density function $$f(x) = \begin{cases} 2\,(x-\mu)e^{-(x-\mu)^2} & \text{if } x > \mu \\ 0 & \text{otherwise,} \end{cases}$$ where $\mu \in (-\infty, \infty)$ is an unknown parameter. It is given that the observed value of $\min\{X_1, X_2, \ldots, X_{10}\}$ is 1. Using the pivot $\min\{X_1, X_2, \ldots, X_{10}\} - \mu$, suppose a 95% confidence interval of $\mu$ is of the form $(c, 1)$, then $c$ equals ______________ *(rounded off to two decimal places).* |

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| Q.34 | Let $X$ be a single observation from a distribution having the probability density function $$f_\theta(x) = \begin{cases} 1 & \text{if } \theta < x < \theta + 1 \\ 0 & \text{otherwise,} \end{cases}$$ where $\theta \in (-\infty, \infty)$. For testing $H_0: \theta \leq 0$ against $H_1: \theta > 1$, let $\beta$ be the power of the uniformly most powerful test of level 0.05. Then 225 $\beta$ equals __________ *(answer in integer).* |
| Q.35 | Let $(X_1, X_2, X_3)^T$ follow a trivariate normal distribution with mean vector $\mu$ and covariance matrix $\Sigma$ given by $$\mu = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$ Then $\mathrm{Var}(X_1 \mid X_2 = 1, X_3 = -1)$ equals __________ *(rounded off to two decimal places).* |

## Q.36 – Q.65 Carry TWO marks Each

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| Q.36 | Consider the function $f: \mathbb{R}^2 \to \mathbb{R}$ defined by $$f(x_1, x_2) = 2x_1^4 + x_2^2 + x_2 x_1^2.$$ Which of the following statements is correct? |
| | |
| (A) | There are two distinct stationary points of $f$ |
| (B) | $(0, 0)$ is a saddle point of $f$ |
| (C) | $(0, 0)$ is a local maximum of $f$ |
| (D) | $(0, 0)$ is a local minimum of $f$ |
| | |

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| Q.37 | Let $x_1 \in (0,4)$ and consider the sequence $\{x_n\}_{n\geq 1}$ defined iteratively by<br>$$x_{n+1} = 2 - (4 - x_n)^{\frac{1}{2}},\ \ n \geq 1.$$<br>Consider the following statements:<br>(I) $\{x_n\}$ converges to 0 .<br>(II) $\left\{\frac{x_{n+1}}{x_n}\right\}$ converges to $\frac{1}{4}$.<br>Which of the following statements is correct? |
| | |
| (A) | Only statement (I) is correct |
| (B) | Only statement (II) is correct |
| (C) | Both statements (I) and (II) are correct |
| (D) | Neither statement (I) nor statement (II) is correct |
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| Q.38 | Let $A \in M_n(\mathbb{R})$ be an $n \times n$ real matrix, $n \geq 2$. Consider the following two statements:<br><br>(I) If $\lambda \in \mathbb{C}$ is an eigenvalue of $A$, then its complex conjugate $\bar{\lambda}$ is also an eigenvalue.<br>(II) If $v = (v_1, v_2, \dots, v_n) \in \mathbb{C}^n$ is an eigenvector corresponding to eigenvalue $\lambda = x + iy,\ y \neq 0$, then $\mathrm{Re}(v) = \big(\mathrm{Re}(v_1), \dots, \mathrm{Re}(v_n)\big)$ and $\mathrm{Im}(v) = (\mathrm{Im}(v_1), \dots, \mathrm{Im}(v_n))$ are linearly independent vectors over $\mathbb{R}$.<br><br>Which of the following statements is correct? |
| | |
| (A) | Only statement (I) is correct |
| (B) | Only statement (II) is correct |
| (C) | Both statements (I) and (II) are correct |
| (D) | Neither statement (I) nor statement (II) is correct |
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| Q.39 | Consider the system of equations<br>$$x + 2y - z = a$$<br>$$x + y + 3z = b$$<br>$$2x + 3y + 2z = c$$<br>Consider the following statements:<br>(I) For every $(a, b, c) \in \mathbb{R}^3$, the above system has a solution.<br>(II) For $(a, b, c) = (0, 0, 0)$, the solution set is given by $$\{(-7t,\ \ 4t,\ \ t) : t \in \mathbb{R}\}.$$<br>Which of the following statements is correct? |
| | |
| (A) | Only statement (I) is correct |
| (B) | Only statement (II) is correct |
| (C) | Both statements (I) and (II) are correct |
| (D) | Neither statement (I) nor statement (II) is correct |
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| Q.40 | Let $(\Omega, \mathcal{F}, P)$ be a probability space, where for $A \subset \Omega,\ A \neq \phi,\ A \neq \Omega$, $\mathcal{F} = \{\Omega, \phi, A, A^c\}$, and $P(\Omega) = 1, P(\phi) = 0, P(A) = \frac{1}{2} = P(A^c)$. Let $X$ and $Y$ be two random variables defined on $\Omega$ as follows: $X(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \in A^c, \end{cases}$ and $Y(\omega) = \begin{cases} 1 & \text{if } \omega \in A^c \\ 0 & \text{if } \omega \in A. \end{cases}$ Then which of the following statements is correct? |
| | |
| (A) | $X$ and $Y$ have the same distribution function |
| (B) | $X = Y$ almost everywhere |
| (C) | $X$ and $Y$ are independent |
| (D) | $E(XY) = 1$ |
| | |

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| Q.41 | Let $X$ and $Y$ be independent and identically distributed geometric random variables having the following probability mass function<br>$$P(X = x) = p(1-p)^x, \quad x = 0, 1, 2, \ldots,$$<br>where $p \in (0,1)$. Then which of the following statements is correct? |
| | |
| (A) | $E(X \mid X+Y) = \frac{X+Y}{2}$ |
| (B) | $P(X = Y) = 1$ |
| (C) | $P(X = Y) = \frac{1-p}{1+p}$ |
| (D) | $E(X \mid X+Y) = \frac{X+Y+2}{2}$ |
| | |

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| Q.42 | Let $X$ be a random variable with support $S = \{0, 1, 2, \dots\}$ and $$P(X \geq k+1 \mid X \geq k) = p, \;\; k \in S, \;\; 0 < p < 1 .$$ Then which of the following statements is correct? |
| | |
| (A) | $P(X \geq k+m \mid X \geq k) = P(X \geq m)$, for all $m, k \in S$ |
| (B) | $E(X) > \mathrm{Var}(X)$ |
| (C) | $P(X \leq x) = 1 - p^x$, for all $x \in S$ |
| (D) | $P(X \leq k+m \mid X \geq k) = 1 - p^m$, for all $k, m \in S$ |
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| Q.43 | Let $X_1, X_2, X_3$ be a random sample from a distribution having probability mass function $$f_\theta(x) = \begin{cases} \theta & \text{if } x = 1 \\ 1-\theta & \text{if } x = 2 \\ 0, & \text{otherwise,} \end{cases}$$ where $\theta \in \Theta = (0,1)$. Let $\underline{X} = (X_1, X_2, X_3)$. Then which of the following is NOT a sufficient statistic for $\theta$? |
| | |
| (A) | $T_1(\underline{X}) = (X_1 - X_2,\ X_1 + X_2,\ X_1 + X_3)$ |
| (B) | $T_2(\underline{X}) = (X_1 + X_2,\ X_1 - X_3,\ X_2 + X_3)$ |
| (C) | $T_3(\underline{X}) = (X_1 - X_2,\ X_2 - X_3,\ X_3)$ |
| (D) | $T_4(\underline{X}) = (X_1 + X_2,\ X_3)$ |
| | |

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| Q.44 | Let $X_1, X_2, \dots, X_n$ $(n \geq 2)$ be a random sample from the probability density function $f(x)$. Consider the following hypotheses: $$H_0: f(x) = \frac{1}{\sqrt{2\,\pi}}\, e^{-\frac{x^2}{2}};\quad -\infty < x < \infty$$ $$H_1: f(x) = \frac{1}{2}\, e^{-\lvert x \rvert};\quad -\infty < x < \infty\,.$$ For testing $H_0$ against $H_1$, let $R$ denote the critical region based on likelihood ratio test having level 0.05. Then, for some constant $c$, the region $R$ is |
| | |
| (A) | $\{(x_1, x_2, \dots, x_n) : \sum_{i=1}^{n} (x_i - 1)^2 > c\}$ |
| (B) | $\{(x_1, x_2, \dots, x_n) : \sum_{i=1}^{n} (x_i - 1)^2 < c\}$ |
| (C) | $\{(x_1, x_2, \dots, x_n) : \sum_{i=1}^{n} (\lvert x_i \rvert - 1)^2 > c\}$ |
| (D) | $\{(x_1, x_2, \dots, x_n) : \sum_{i=1}^{n} (\lvert x_i \rvert - 1)^2 < c\}$ |

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| Q.45 | Let $X_1, X_2, \dots, X_n$ $(n > 1)$ be a random sample from the following probability density function $$f_\beta(x) = \begin{cases} \beta e^{-x}\,(1 - e^{-x})^{\beta-1} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$ where $\beta > 0$ is an unknown parameter. For testing the following hypotheses, $$H_0: \beta = 1 \quad \text{against} \quad H_1: \beta > 1,$$ at level $\alpha \in (0, 1)$, which of the following statements is correct? |
| | |
| (A) | The uniformly most powerful test does not exist |
| (B) | For some constant $a$, the critical region of the uniformly most powerful test will be of the form $C = \{(x_1, x_2, \dots, x_n): \sum_{i=1}^{n} x_i > a\}$ |
| (C) | For some constant $a$, the critical region of the uniformly most powerful test will be of the form $C = \{(x_1, x_2, \dots, x_n): \sum_{i=1}^{n} \ln(1 - e^{-x_i}) > a\}$ |
| (D) | For some constant $a$, the critical region of the uniformly most powerful test will be of the form $C = \{(x_1, x_2, \dots, x_n): \sum_{i=1}^{n} \ln(1 - e^{-x_i}) < a\}$ |
| | |

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| Q.46 | Let $X_1, X_2, \dots, X_5$ be random observations from a continuous distribution. Let $\theta_p$ be $p$-th population quantile. Consider the following hypotheses<br>$$H_0: \theta_{\frac{1}{2}} = 2.5 \quad \text{against} \quad H_1: \theta_{\frac{1}{2}} > 2.5.$$<br>Let $X_{(r)}$ denote the $r$-th order statistic of the given observations. Then which of the following is a critical region of a level 0.05 test? |
| | |
| (A) | $X_{(1)} > 2.5$ |
| (B) | $X_{(2)} > 2.5$ |
| (C) | $X_{(3)} > 2.5$ |
| (D) | $X_{(4)} > 2.5$ |
| | |

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| Q.47 | Suppose that $X$ and $Y$ are independent and identically distributed $N_p(\mu, \Sigma)$ random vectors, where $\mu \in \mathbb{R}^p$ and $\Sigma$ is a positive definite matrix. Let $\chi_m^2$ denote chi-square distribution with $m$-degrees of freedom. Then which of the following statements is correct? |
| | |
| (A) | $\frac{1}{2}\,(X-Y)^T\Sigma^{-1}(X-Y)$ follows $\chi_p^2$ |
| (B) | $2\,(X-Y)^T\Sigma\,(X-Y)$ follows $\chi_p^2$ |
| (C) | $\frac{1}{2}\,(X-Y)^T\Sigma^{-1}(X-Y)$ follows $\chi_{2p}^2$ |
| (D) | $2\,(X-Y)^T\Sigma\,(X-Y)$ follows $\chi_{2p}^2$ |
| | |

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| Q.48 | Consider the multiple linear regression model<br><br>$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \epsilon_i, \;\; i = 1, 2, \dots, 31,$$<br><br>where $\epsilon_i$ are independent and identically distributed $N(0,1)$ variables. The $F$-test for testing significance of regression rejects the null hypothesis $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ against the alternative $H_A$ : at least one $\beta_j$ is not equal to 0, $j = 1, 2, 3,$ at 5% level of significance. It is given that<br>$F_{0.05;\,3,\;27} = 2.96,\;\; F_{0.05;\,3,\;30} = 2.92,\;\; F_{0.025;\,3,\;27} = 4.01,\; F_{0.025;\;3,\;30} = 3.91,$<br>where, for a random variable $F$ following central $F$-distribution with $(m, n)$ degrees of freedom, $P\left(F > F_{\alpha;\,m,n}\right) = \alpha, \alpha \in (0,1)$. Then the value of $R^2$ cannot be equal to |
| | |
| (A) | 0.50 |
| (B) | 0.80 |
| (C) | 0.30 |
| (D) | 0.20 |
| | |

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| Q.49 | Consider the function $f: \mathbb{R}^2 \to \mathbb{R}$ defined by $$f(x,y) = \begin{cases} \dfrac{x^3 + y^3}{\sqrt{x^2 + 2y^2}} & (x,y) \neq 0 \\ 0 & (x,y) = 0. \end{cases}$$ Which of the following statements is/are correct? |
| | |
| (A) | $f$ is continuous at $(0,0)$ |
| (B) | Partial derivatives $f_x$ and $f_y$ exist at $(0,0)$ and $f_x(0,0) = 0,\ f_y(0,0) = 0$ |
| (C) | $f$ is not differentiable at $(0,0)$ |
| (D) | $f$ is differentiable |
| | |

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| Q.50 | Consider the matrix<br>$$A = \begin{pmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_2 & 1 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \qquad \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}.$$<br>Which of the following statements is/are correct? |
| | |
| (A) | $A$ is never diagonalizable |
| (B) | $A$ is diagonalizable if and only if $\lambda_1, \lambda_2, \lambda_3$ are distinct |
| (C) | The dimension of eigenspaces corresponding to each $\lambda_i$ is $1, i = 1, 2, 3$ |
| (D) | For any $A$, there exist real numbers $\alpha$ and $\beta$ such that $A^2 + \alpha A + \beta I_3 = 0$ |
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| Q.51 | Consider the real symmetric matrix $A = (a_{ij})$ given by $$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}.$$ Consider the set $$S = \left\{ x = (x_1, x_2, x_3) \in \mathbb{R}^3 : \sum_{j=1}^{3} \sum_{i=1}^{3} a_{ij} x_i x_j = 1 \right\}.$$ Which of the following statements is/are correct? |
| | |
| (A) | $S$ is empty set |
| (B) | Any line $L$ in $\mathbb{R}^3$ has at most two points from $S$ |
| (C) | $S$ is bounded |
| (D) | $S$ is unbounded |
| | |

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| Q.52 | Let $X$ be a discrete random variable with support $S = \{1,2,3,\dots\}$ such that $E(X^2) < \infty$. Let $F$ be the distribution function of $X$. Then which of the following statements is/are correct? |
| | |
| (A) | $E(X) = \sum_{n=1}^{\infty}(1 - F(n-1))$ |
| (B) | $F(X)$ has discrete uniform distribution |
| (C) | There exists at least one such random variable $X$ such that $X$ and $\frac{1}{X}$ have the same distribution |
| (D) | $E(X^2) = \sum_{n=1}^{\infty}(2n-1)\big(1 - F(n-1)\big)$ |
| | |

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| Q.53 | Let $\{X_n\}_{n\geq 1}$ be a sequence of random variables having the following probability mass function $$P(X_n = x) = \frac{1}{5n}\left(1 - \frac{1}{5n}\right)^x, \quad x = 0, 1, 2, \ldots;\ n \in \mathbb{N}.$$ Define $Z_n = \frac{X_n}{n}$, $n \in \mathbb{N}$, and let $V$ be a random variable. If $Z_n \xrightarrow{d} V$, as $n \to \infty$, then which of the following statements is/are correct? |
| | |
| (A) | $V$ has normal distribution with mean 5 and variance 25 |
| (B) | $V$ has chi-square distribution with 5 degrees of freedom |
| (C) | $V$ has exponential distribution with mean 5 |
| (D) | $e^{-\frac{V}{5}}$ has uniform distribution over $(0, 1)$ |
| | |

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| Q.54 | Let $\{X_k\}_{k\geq 1}$ be a sequence of independent random variables such that $$X_{2k-1} \sim \text{Bin}(1, \theta), \text{ and } \quad X_{2k} \sim \text{Bin}(1, 1-\theta), \ \ k = 1,2,3, \dots,$$ where $\theta \in (0, 1)$. Let $\{Y_k\}_{k\geq 1}$ be another sequence of independent and identically distributed random variables such that $Y_k \sim \text{Poisson}(\lambda), \lambda > 0$. Define, for $n \in \mathbb{N}$, $$S_{2n} = \sum_{k=1}^{n} (X_{2k-1} - X_{2k} + 1 - 2\theta), \quad W_n = \sum_{k=1}^{n} Y_k^2 \quad \text{and} \quad \sigma_{2n}^2 = 2n\theta(1-\theta).$$ Then which of the following statements is/are correct? |
| | |
| (A) | $\frac{n\, S_{2n}}{\sigma_{2n} W_n} \xrightarrow{d} N(0, 1)$, as $n \to \infty$ |
| (B) | $\frac{n\, S_{2n}}{\sigma_{2n} W_n} \xrightarrow{d} N\left(0, \frac{1}{\lambda^2(1+\lambda)^2}\right)$, as $n \to \infty$ |
| (C) | $\frac{n\, S_{2n} + \sigma_{2n} W_n}{n\, \sigma_{2n}} \xrightarrow{d} N(\lambda(1+\lambda), 1)$, as $n \to \infty$ |
| (D) | $\frac{n\, S_{2n} + \sigma_{2n} W_n}{n\, \sigma_{2n}} \xrightarrow{d} N\left(0, \frac{1}{\lambda^2(1+\lambda)^2}\right)$, as $n \to \infty$ |
| | |

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| Q.55 | Let $\{N(t); t \geq 0\}$ be a homogeneous Poisson process with rate 3, and let $T_1$ denote the first arrival time. Then which of the following statements is/are correct? |
| | |
| (A) | $E(T_1) = 3$ |
| (B) | $E(T_1) = \frac{1}{3}$ |
| (C) | $E(T_1 \mid N(2) = 4) = \frac{6}{5}$ |
| (D) | $E(T_1 \mid N(3) = 4) = \frac{3}{5}$ |
| | |

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| Q.56 | Let $X_1, X_2, \dots, X_n$ $(n \geq 2)$ be a random sample from the following probability density function $$f(x) = \frac{1}{2} e^{-\lvert x-\mu \rvert}, \quad -\infty < x < \infty,$$ where $\mu \in (-\infty, \infty)$ is an unknown parameter. Let $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ and $\hat{\mu}$ denote the maximum likelihood estimator of $\mu$, whenever it exists. Then which of the following statements is/are correct? |
| | |
| (A) | $\hat{\mu}$ will always exist and $\hat{\mu} = \bar{X}$ |
| (B) | $\hat{\mu}$ may not always exist |
| (C) | $\hat{\mu}$ always exists but it may not be unique |
| (D) | $\hat{\mu}$ is a consistent estimator of $\mu$ |
| | |

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| Q.57 | Let $X$ be a single observation from a distribution having a probability density function $f_\theta,\ \theta \in \Theta$. For testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$, under which of the following options a uniformly most powerful test of level 0.05 exists? |
| | |
| (A) | $f_\theta(x) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$ ; $\Theta = (0, \infty)$ |
| (B) | $f_\theta(x) = \begin{cases} e^{-(x-\theta)} & \text{if } x > \theta \\ 0 & \text{otherwise} \end{cases}$ ; $\Theta = (-\infty, \infty)$ |
| (C) | $f_\theta(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$ ; $\Theta = (0, \infty)$ |
| (D) | $f_\theta(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x-1}{\theta}} & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases}$ ; $\Theta = (0, \infty)$ |
| | |

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| Q.58 | Suppose that $X = (X_1, \dots, X_p)^T$ follows $N_p(\mu, \Sigma)$, where $\mu \in \mathbb{R}^p$ and $\Sigma$ is a positive definite matrix. Let $A$ be a $p \times p$ matrix such that $A^T A = I_p$.<br><br>Let $Y = (Y_1, \dots, Y_p)^T = AX$. Then which of the following statements is/are correct? |
| | |
| (A) | $Y$ follows $N_p(A\mu,\ A^T\Sigma A)$ |
| (B) | $Y$ follows $N_p(A\mu,\ A\Sigma A^T)$ |
| (C) | $\mathrm{Var}\left(\sum_{i=1}^{p} Y_i\right) = \mathrm{Var}\left(\sum_{i=1}^{p} X_i\right)$ |
| (D) | $\sum_{i=1}^{p} \mathrm{Var}(Y_i) = \sum_{i=1}^{p} \mathrm{Var}(X_i)$ |
| | |
| Q.59 | Suppose that $X = (X_1, \dots, X_p)^T$ follows $N_p(0, \Sigma)$, where $\Sigma$ is a positive definite matrix. Then which of the following statements is/are correct? |
| | |
| (A) | $X_1, \dots, X_p$ are always independent normal random variables |
| (B) | $X_1, \dots, X_p$ are normal random variables |
| (C) | $X_1^2 + \cdots + X_p^2$ always follows a chi-square distribution |
| (D) | Any linear combination of $X_1, \dots, X_p$ is a normal random variable |
| | |

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| Q.60 | Let $Y = (Y_1, Y_2, Y_3)^T \sim N_3(0, I_3)$, where $I_3$ denotes the identity matrix of order 3. Let $$A = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{and} \quad B = I_3 - A.$$ Let $\chi_1^2$ denote chi-square distribution with 1 degree of freedom. Then which of the following statements is/are correct? |
| | |
| (A) | $Y^T AY \sim \chi_1^2$ |
| (B) | $Y^T BY \sim \chi_1^2$ |
| (C) | $Y^T AY$ and $Y_1 - 2Y_2 + Y_3$ are independently distributed |
| (D) | $Y^T AY$ and $Y_1 + 2Y_2 + Y_3$ are independently distributed |
| | |
| Q.61 | Let $D = \{(x, y) \in \mathbb{R}^2 : 0 < x^2 + y^2 \leq 1\}$. For $\alpha \geq 0$, consider the integral $$I_\alpha = \iint_D \frac{1}{(x^2 + y^2)^\alpha} \, dx\, dy.$$ Let $$N_0 = \sup\{\alpha \geq 0 : I_\alpha < \infty\}.$$ Then the value of $N_0$ equals _______________ *(answer in integer).* |

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| Q.62 | Let $X$ follow $N(3,1)$. Then the value of $E(X^4(X-3))$ equals ______________ *(answer in integer).* |
| Q.63 | Let $U_1, U_2, U_3$ be three independent exponential random variables such that $U_k$ has the following probability density function $$f_k(x) = \begin{cases} k\, e^{-kx} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases} \qquad k = 1, 2, 3.$$ Let $$X = \min\{U_1, U_3\} \quad \text{and} \quad Y = \min\{U_2, U_3\}.$$ Then $P(X = Y)$ equals ______________ *(rounded off to two decimal places).* |
| Q.64 | Let $X_1, X_2$ be a random sample from a distribution having the population density function $$f(x) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x < \theta \\ 0 & \text{otherwise,} \end{cases}$$ where $\theta \in (0, \infty)$. Let $X_{(2)} = \max\{X_1, X_2\}$ and $$\psi(\theta) = P_\theta(X_1 + X_2 < 1), \ \ \theta > 0.$$ Let $\delta(X_{(2)})$ be an unbiased estimator of $\psi(\theta)$ that depends on observations $X_1$ and $X_2$ only through $X_{(2)}$. If $\delta(t)$ is a continuous function on $(0, \infty)$, then the value of $18\,\delta\left(\frac{3}{4}\right)$ equals ______________ *(answer in integer).* |

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| Q.65 | Let $X$ be a single observation from a distribution having probability density function<br><br>$$f_\theta(x) = \begin{cases} \frac{2x}{\theta^2} & \text{if } 0 < x < \theta \\ 0 & \text{otherwise,} \end{cases}$$<br><br>where $\theta \in (0, \infty)$. For testing $H_0: \theta \leq 1$ against $H_1: \theta > 1$, at level of significance 0.05, let $\beta_1$ be the size of the uniformly most powerful test and $\beta_2$ be the power of the uniformly most powerful test at $\theta = 2$. Then $10\,\beta_1 + 40\,\beta_2$ equals _______________ *(answer in integer)*. |